Structured Problem-Solving and Extensive Discussion (Neriage) Approach in Teaching Mathematics: A Case Study on Hokkaido University Education's Attached Schools

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算数指導における構造化された問題解決と練り上げアプローチ

― 北海道教育大学附属学校の事例研究 ―

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ABSTRACT

The Japanese approach in teaching mathematics is quite different from other countries. Since Japan is one of the top-ranked countries in mathematics teaching and learning, many educational organizations are interested in its system. This study examines the structured problem-solving approach, which is centered on the strategy of extensive discussion (Neriage), and investigates how these strategies are planned and executed by teachers at the schools attached to Hokkaido University of Education. How these approaches contribute to the effective achievement of goals in the classroom and national standards is also analyzed. Additionally, mathematics textbooks were analyzed and classroom observations were conducted. A survey was conducted to examine the facilitation and effective use of the approaches. Findings from the different analyses and observations indicate that Neriage is a critical part of the structured problem-solving approach, which is a significant factor in the achievement of learning goals and development of higher-order mathematics skills among students.

1. Introduction

1. 1. Research Background

Teaching mathematics is one of the greatest challenges among educators around the world. Different approaches are used by different countries, educational institutions, or even teachers because students have different learning styles, and there cannot always be a one-size-fits-all approach when it comes to teaching mathematics.

Meanwhile, Japanese educators use a unique approach in teaching mathematics, which seemed to be very effective based on Japan's high ranking in 2018 OECD's Programme for International Student Assessment (PISA) for mathematics literacy. [1]

"Structured Problem-Solving," as described by Stigler and Hiebert in *The Teaching Gap*, has been an interest in the West. This Japanese problem-solving approach is different from how problem-solving is taught in many countries. According to Takahashi (2006), a famous Japanese researcher and educator, the Japanese problem-solving lesson devotes substantial time to students devising their own ways of solving a problem, and this is seen as preparation for the crucial *Neriage* phase of the lesson, in which the teacher leads an extended plenary discussion and different solution methods are shared and compared. This is far from other countries' approaches, which are usually focused on the process of solving a problem and not necessarily focused on developing mathematical concepts and skills. [2] These problem-solving lessons usually employ a question-and-answer type of exposition, where the teacher discusses most of the rules and processes in solving a problem and students simply listens to the teacher and copies the process that the teacher presented.

The structured problem-solving approach in Japan is used throughout the country by all the learning institutions from early grade levels until the higher ones, training the students to be problem solvers rather than only concept absorbers. This approach attracted much attention and recognition from all over the world.

1. 2. Statement of the Problem

This research focuses on the structured problem-solving and extensive discussion (*Neriage*) approach. To obtain all the essential knowledge, data, and information, the research sought to answer the crucial questions as follow:

1. How do math teachers usually select/ construct a word problem for a specific lesson?
2. What are the instructional materials usually used by math teachers in presenting word problems?
3. How do math teachers facilitate the extensive discussion (*Neriage*) approach in each math strand?
4. How do math teachers facilitate the structured problem-solving approach?
5. How does *Neriage* support productive achievement of problem-solving tasks?

1. 3. Significance of the Study

The findings of this study will benefit constructivism learning theory since the structured problem-solving and *Neriage* approach play an important role in efficient mathematics education in Japan. The direct recipients of the output of this research are student. Any improvement of mathematics teaching strategies can pave the way to improve the student learning experience. Moreover, this study will be very beneficial to the teachers and

education practitioners who have not used the stated approaches.

2. Review of Related Literature

2. 1. Mathematics Lesson

A major reform movement in teaching and learning mathematics occurred during the 1970s and 1980s where traditional classrooms that focused on teachers' instruction were shifted to student-centered classrooms that focus on students' engagement in mathematical activities. Using the Trends in International Mathematics and Science Study videotaped classroom study, Stigler and Hiebert argue that Japanese mathematics lessons better exemplify current U.S. reform ideas than do typical U.S. mathematics lessons. [3]

Generally, mathematics lessons in Japan can be characterized as following a problem-solving approach: the teachers emphasize the students' problem-solving and attempt to teach the meaning and use of mathematical concepts by reflecting on solutions. A school hour lasts 45 minutes at elementary and 50 minutes at junior high school. Students are allotted 20 to 30 minutes for discussion of solutions. Content related to a single mathematical topic typically comprises a teaching unit of about 10 hours. [4]

In general, Japanese mathematics lessons include the following characteristics:

1. Student-centered instruction using problem-solving as foundation
2. Structured problem-solving
3. Carefully selected word problems and activities, and their cohesiveness
4. *Neriage*
5. Emphasis on blackboard practice (*Bansho*)

Teachers appear to take a less active role, allowing their students to invent their own procedures for solving problems, which are quite demanding, both procedurally and conceptually. Teachers, however, carefully design and orchestrate lessons so that students are likely to use procedures that have been developed recently in class. Thus, the selection of a problem for the problem-solving activity in each class is extremely critical for teachers when they plan a lesson. [3]

2. 2. Structured Problem-Solving

Teaching methods were developed differently in Japan compared to other countries. Hiebert, Stigler, and Manaster (1999) argue that Japanese teachers emphasize mathematical thinking rather than mathematical skills. This goal is reached by having the students discuss the possible solutions to the problems presented to the whole class with the teacher and peers. [5] In this structure, teachers introduce new mathematical concepts by giving students new problems to try to solve their own. The teacher does not act as a lecturer but instead observes students as they work independently and facilitates a whole class discussion of their solution approaches. (Fig. 1) [6]

Major characteristics of these problem-solving lessons include:

a. *Hatsumon*: the thought-provoking question or problem that students

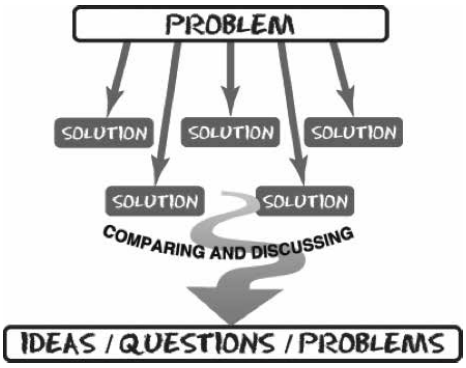


Fig. 1 Structured Problem-Solving (Takahashi, 2006)

engage with and that is the key to students' mathematical development and mathematical connections

b. *Kikan-shido:* sometimes referred to as the "purposeful scanning" that takes place while students are working individually or in groups, which allows teachers not only to monitor students' strategies but also to orchestrate their reports on their solutions in the *Neriage* phase of the lesson

c. *Neriage*: the "kneading" stage of a lesson that allows students to compare, polish, and refine solutions through the teacher's orchestration and probing of student solutions

d. *Matome*: the summing up and careful review of students' discussion to guide them to higher levels of mathematical sophistication [7]

This instructional approach emphasizes the process of problem-solving activities and provides students with opportunities to re-invent mathematical ideas and concept by themselves. It is designed to create interest in mathematics and stimulate creative mathematical activity in the classroom through students' collaborative work. [3]

2. 3. Extensive Discussion (Neriage)

In the structured problem-solving approach, the teacher presents a problem to students without giving a procedure, and it is natural that several different approaches to the solution will come from the students.

Neriage describes the dynamic and collaborative nature of a whole class discussion in the lesson (Shimizu, 1999). *Neriage* begins after the students present their various solution methods. Until *Neriage* begins, the whole class activity is very similar to the children's favorite school activity, Show and Tell. However, *Neriage* is an activity that goes beyond Show and Tell. Teachers might begin *Neriage* by asking students to see if there are some common ideas or approaches among the solution methods. Then, teachers would lead students to see whether each approach has advantages and limitations. *Neriage* is a critical component of a Japanese problem-solving lesson because this is where teachers can teach students new mathematical ideas and concept by using the students' solution methods. Because *Neriage* is built upon the students' solution as a foundation of the dynamic and collaborative whole class discussion, Japanese teachers put extensive effort into preparing the discussion. [8]

The structure of *Neriage* can be flexible depending upon the topic of the lesson, the nature of the problem, and the objective of the lesson. There are four main types of *Neriage,* each depending on the needs of the lesson and the classroom situation.

a. Type 1 *Neriage:* Develop a New Idea by Examining the Progression of Thought (Fig. 2)

This type introduces students' various approaches in order from simplest to most sophisticated.

By starting the discussion with a method that every student understands, the teacher will be able to lead students through successively more sophisticated strategies so that by the end, nearly every student will achieve the lesson goal. This type of *Neriage* allows students to begin the discussion on a level that everyone understands. The teacher's role is to carefully sequence the students' work, making sure

everyone can follow along.

b. Type 2 *Neriage*: Compare Multiple Approaches to Learn a New Concept (Fig. 3)

This type compares solution side by side to uncover what they have in common.

c. Type 3 *Neriage*: Address Misconceptions Through Debate (Fig. 4)

Misconceptions are opportunities to help students look back and solidify their understanding of concepts. Type 3 *Neriage* utilizes a task designed to provoke both correct

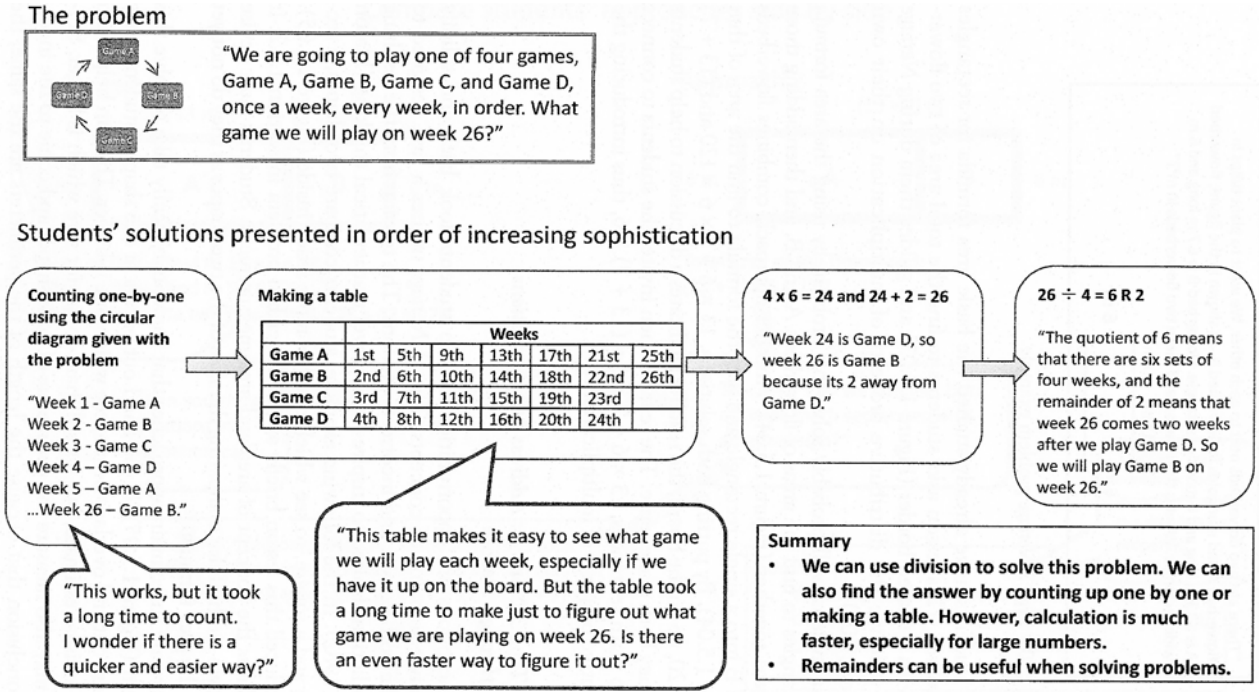


Fig. 2 Type 1 *Neriage* Sample

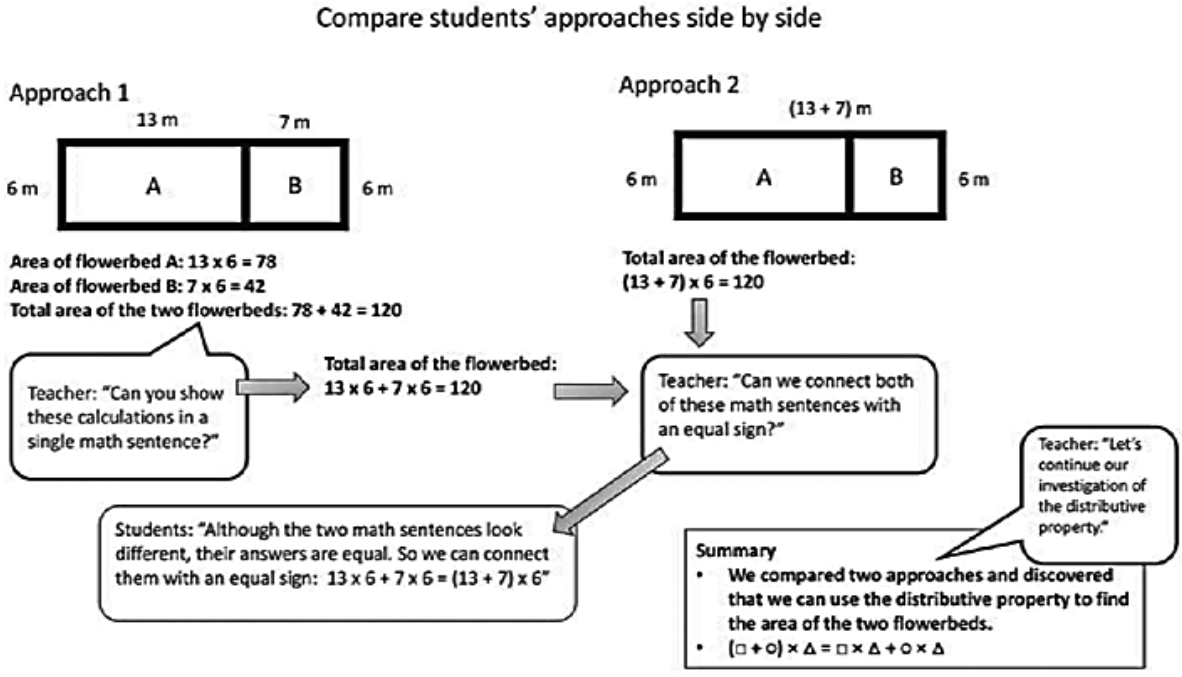


Fig. 3 Type 2 *Neriage* Sample

and incorrect solutions. The tension from the resulting contradictions is used to inspire higher-level mathematical thinking.

d. Type 4 *Neriage*: Compare Various Solutions to an Open-Ended Problem (Fig. 5)

The especially fluid and dynamic Type 4 *Neriage* compares students' various solutions to open-ended problems for which there are multiple correct solutions. [9]

2. 4. Blackboard Practice (*Bansho*)

Japanese classrooms nearly always contain at least one large blackboard that stretches across the entire width of the classroom, and this provides very particular opportunities for structuring the *Neriage* phase of the Japanese problem-solving lesson by showcasing multiple solution methods side by side, for comparison and discussion. *Bansho*, "the intentional use of board space for facilitating student learning," is a highly developed skill, and planning for

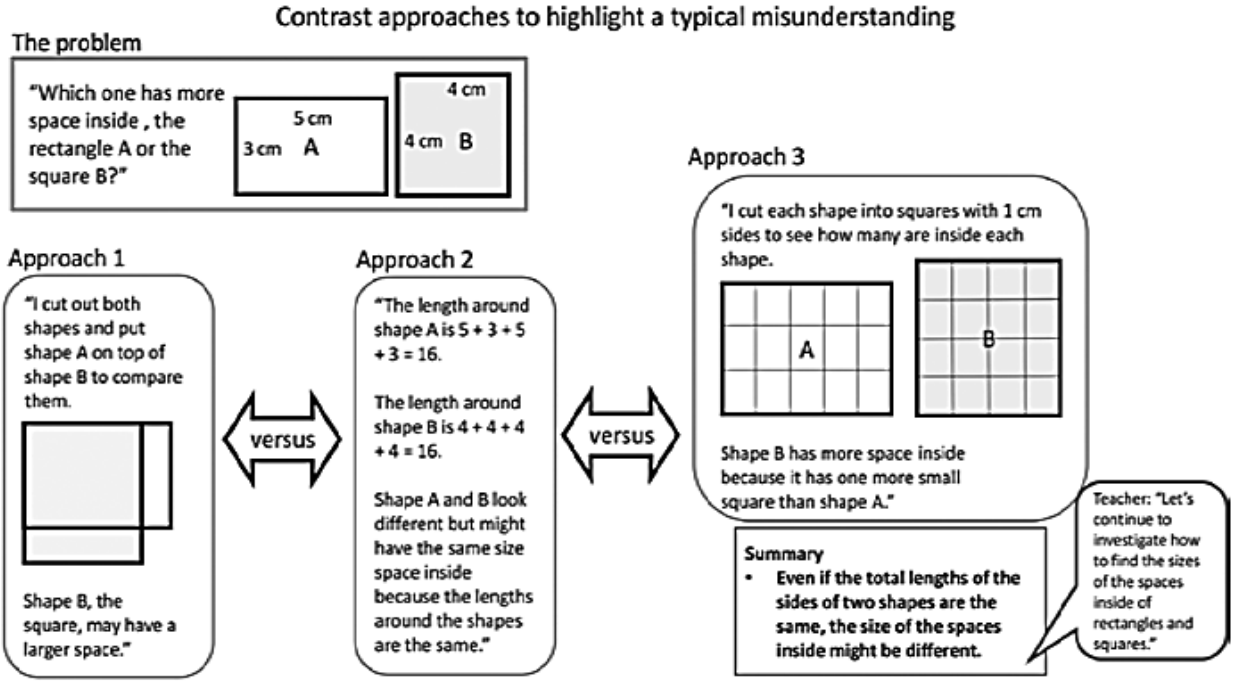


Fig. 4 Type 3 *Neriage* Sample Lesson

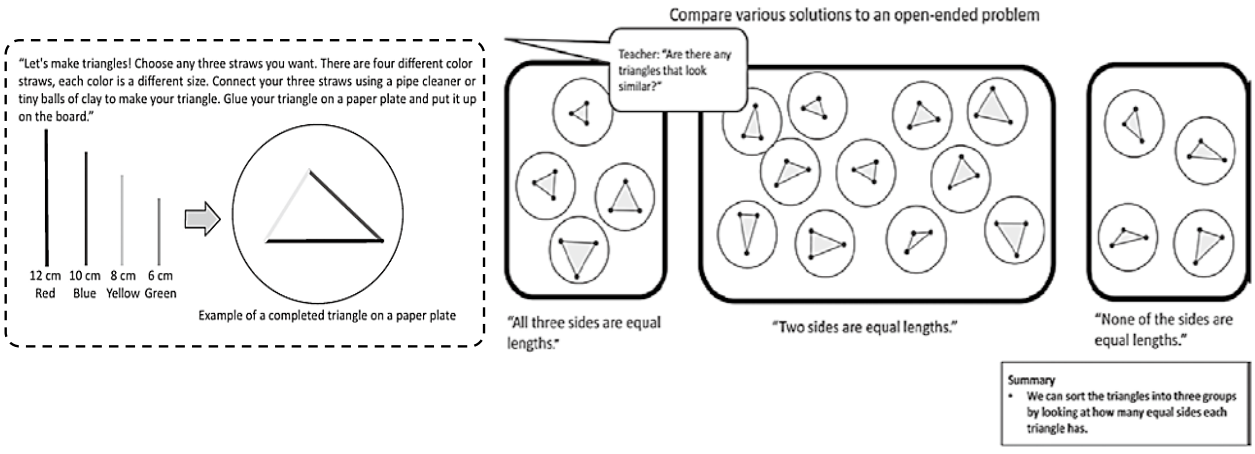


Fig. 5 Type 4 *Neriage* Sample

effective use of the board is an important part of teachers' professional development. *Bansho Keikaku* (boardwork planning) is central to lesson planning and includes consideration of the lesson content, the resources being used, and likely student responses. (Fig. 6) [2]

The following are the uses of *Bansho*.

- To keep a record of the lesson
- To help students remember what they need to do and to think about
- To help students see the connection between different parts of the lesson and the progression of the lesson
- To compare, contrast, and discuss ideas that students present
- To help organize student thinking and discovery of new ideas
- To foster organized student note-taking skills by modeling good organization [3]

3. Research Methodologies

3. 1. Research Instrument

The following methodologies are used to accomplish the objectives of this research.

a. Textbook Analysis: math textbooks used by the schools were identified and analyzed. The content and the flow of the math textbooks were also analyzed and described in this research.

b. Lesson Observation: math lessons and the teacher-student interactions specifically at the schools attached to Hokkaido University of Education were observed from May to December 2021. From this observations, the facilitation of structured problem-solving and *Neriage* were analyzed and described.

c. Questionnaire: All math teachers from schools attached to Hokkaido University of Education were asked to complete a questionnaire related to their strategies in facilitating a structured problem-solving approach and *Neriage*.

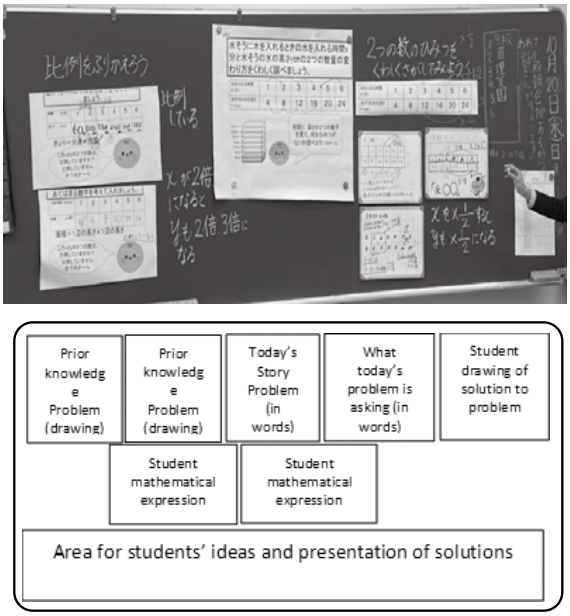


Fig. 6 A *Bansho* during a Math Class at Fuzoku (top) and a sample *Bansho* plan (bottom)

3. 2. Statistical Treatment Applied

The following were used to analyze questions #1 and #2 of the questionnaire.

a. Putting Weight for the Response: Since the respondents ranked their answers, a weight is put to each response. For the 1st option, the weight is 1. 2 for the 2nd option, 3 for the 3rd and so on.

b. Weighted Average: Since the respondents ranked their answers, weighted average of each option was computed.

c. Rank: After solving for the average of each response, the weighted averages were ranked to see which option is most frequently used by the respondents. Since the weight of 1 is put for the 1st option, the weighted average closest to 1 is the most-used option.

For question number 3, the Percentage

of the responses for each *Neriage* type was computed to see which is commonly used by the respondents.

Qualitative analysis of data, which refers to the process of categorizing verbal or behavioral data to classify, summarize, or tabulate data, was used to describe the results in questions #4, #5, textbook analysis, and lesson observations. [10]

4. Presentation, Analysis, and Interpretation of Data

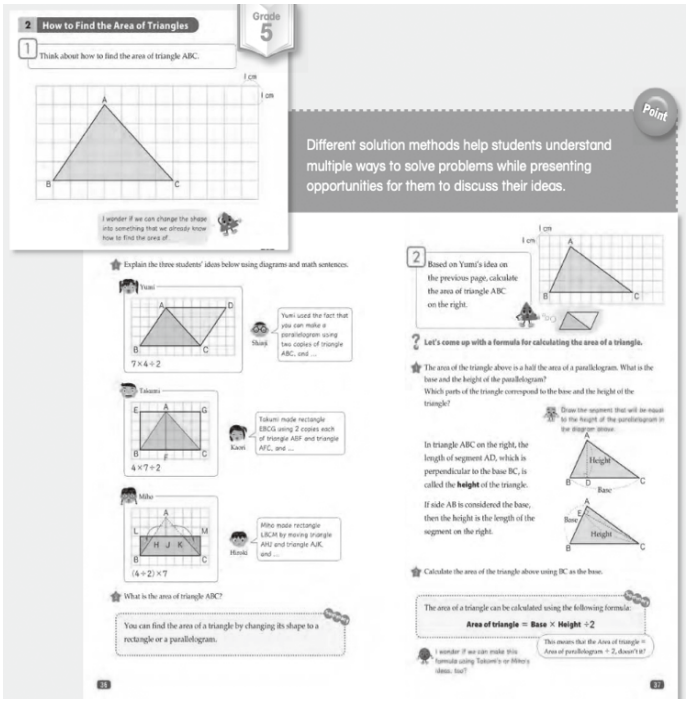
4. 1. Textbook Analysis

Two sets of textbooks from two different publishers in Japan were analyzed in this study: Keirinkan co.ltd's *Fun With Math* and Tokyo shoseiki's *Mathematics International*. (Fig. 7) The textbook analyses revealed the following.

1. Mathematics textbooks include open-ended questions to guide students in developing their understanding, instead of explanations of concepts and procedures.
2. The book series contain more alternative approaches for solving a problem and provide illustrations and pictures to help students solve problems independently.
3. The flow of the presentation of the lesson in the textbook is the same as the teachers' presentation in class, providing multiple approaches to the word problems.
4. Students are encouraged to solve problems, present and explain their ideas, understand different solution strategies, and identify and generate key mathematical ideas by providing story problems that students could answer independently and by providing helpful hints and highlighting important points.

4. 2. Lesson Observation Analysis

Math lessons and the teacher-student interactions specifically at the schools attached to the Hokkaido University of Education were



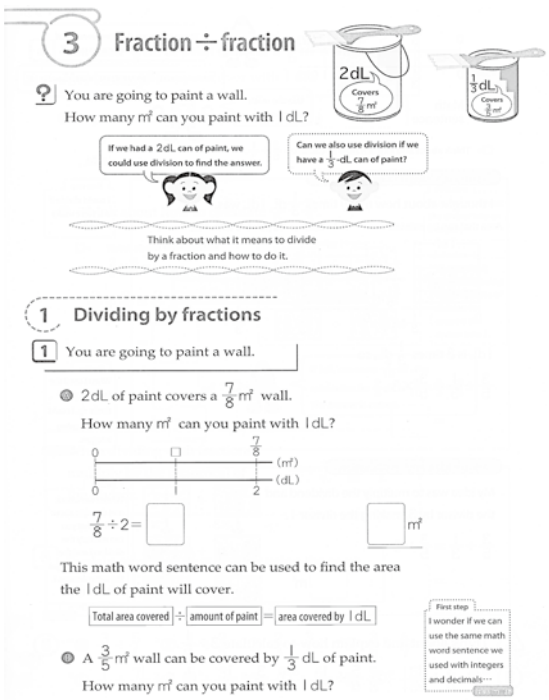


Fig. 7 Parts of a Math Textbook in Japan. Tokyo shoseki's *Mathematics International* (left, retrieved from www.docplayer.net) and Keirinkan co.ltd's *Fun With Math*

observed from May to December 2021. (Fig. 8) From the 18 lesson observations, the following details were identified.

1. The teachers used a common and consistent flow in teaching the students problem-solving. Each lesson included starting from the review of the past lesson, presenting the problem of the day, conducting individual and group thinking, facilitating *Neriage*, and then generalization. This flow is done in every day's lesson.
2. Even though the teacher gave 1-3 word problems a day, the focus was still within the same topic, and the students still provided multiple solutions to the problems.
3. There was more time provided for individual thinking and group discussion to give way for deeper analysis, discussion, and production of multiple solutions.
4. During *Neriage*, the students were the ones who explained solutions. The whole class decided if the solutions presented were valid and accurate. The lesson usually ended by identifying all the possible approaches to the problem of the day and applying it to other situations.
5. In the 18 observations, the type 4 *Neriage* was the usual approach used by the teachers.
6. Although there was a common approach used, other strategies such as games and ICT integration were still used.

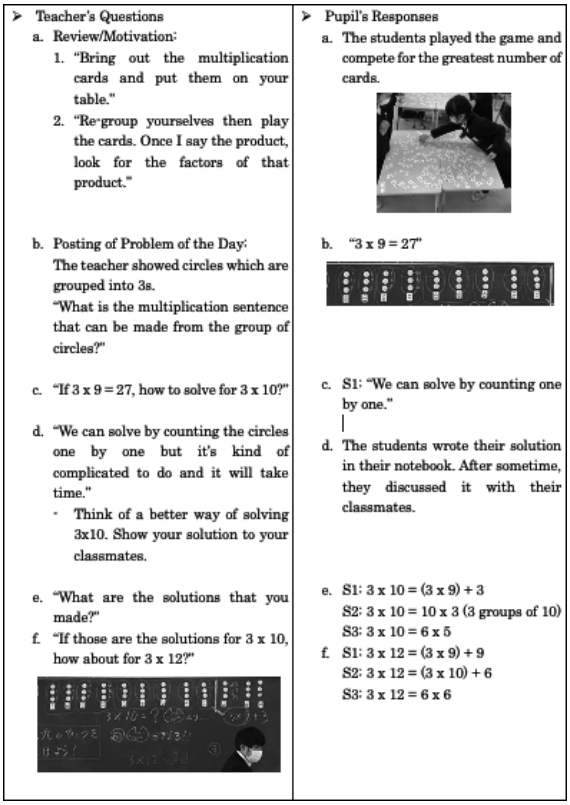


Fig. 8 Sample Lesson Observation at Fuzoku Elementary School on Dec. 10, 2021

4. 3. Questionnaire Analysis

All math teachers from the schools attached to the Hokkaido University of Education were asked to complete a questionnaire related to their strategies in facilitating a structured problem-solving approach and *Neriage*. From the survey, the following responses were identified.

1. How do math teachers usually select/ construct a word problem for a specific lesson?

Table 1 Common Sources of Word Problems

Responses	WA	Rank
Textbook	1	1
Online Resources	3.090909	2
Lesson Study Booklet	3.545455	4
Previous Lesson Plans	3.909091	5
Self-Creation	3.454545	3

Table 1 shows that the most frequently used source of word problems for math teachers is "Textbook," having a rank of 1. This is followed by "Online Resources" in the 2nd rank,

then "Self-Creation" in the 3rd rank. The 4th rank is "Lesson Study Booklet" followed by "Previous Lesson Plans."

2. What are the instructional materials usually used by the math teachers in facilitating structured word problem approach?

Table 2 Commonly Used Instructional Materials

Responses	WA	Rank
Pictures	1.727273	1
Manipulatives/Real Objects	3.090909	3
ICT Tools/Software	2.363636	2
Videos/Songs	4.636364	5
Worksheet/Problem Set	3.181818	4

Table 2 shows that the most frequently used instructional materials that math teachers used are "Pictures" having a rank of 1. This is followed by "ICT Tools/Software" in the 2nd rank, "Manipulatives/Real Objects" in the 3rd rank, and "Worksheet/Problem Set" and "Videos/Songs" in 4th and 5th ranks, respectively.

3. How do math teachers facilitate the *Neriage* approach in each math strand?

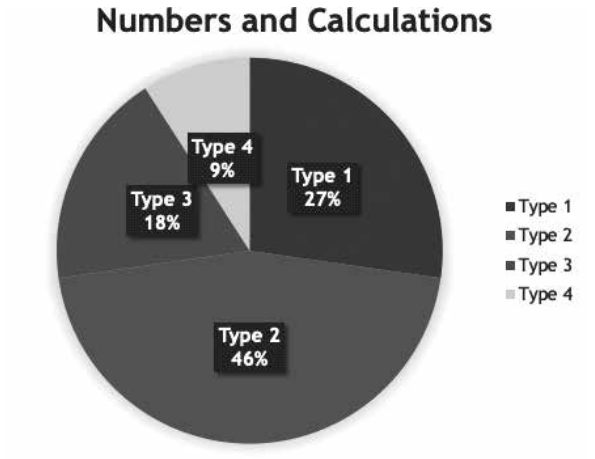


Fig. 9 Numbers and Calculations

The figure 9 shows that 46% of the respondents use Type 2 *Neriage* in teaching Numbers and Calculations. Type 1 is used by 27%, 18% use Type 3, and 9% use Type 4.

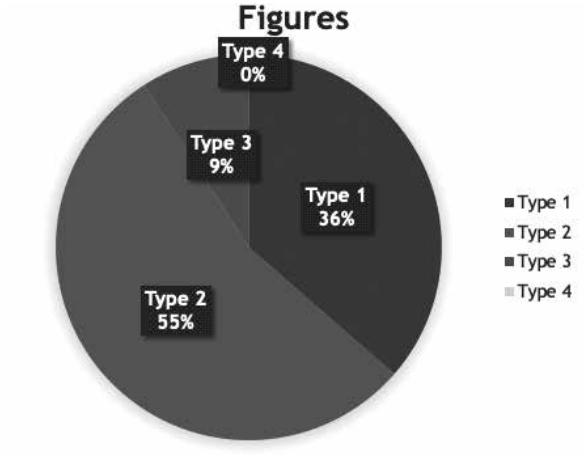


Fig. 10 Figures

The figure 10 shows that 55% of the respondents use Type 2 of *Neriage* in teaching Figures. Type 1 is used by 36%, 9% use Type 3, and 0% use Type 4.

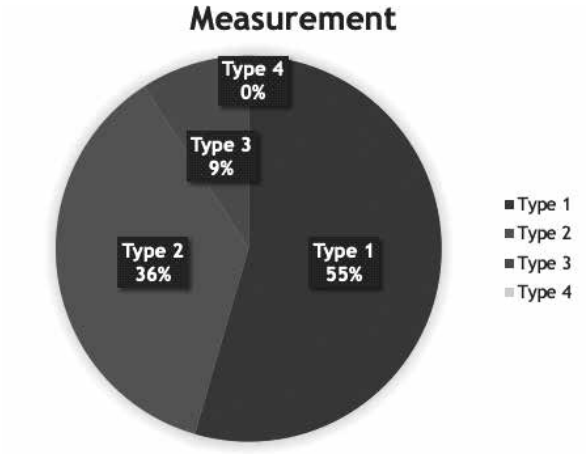


Fig. 11 Measurement

The figure 11 shows that 55% of the respondents use Type 1 of *Neriage* in teaching Measurement. 36% use Type 2, 9% use Type 3, and 0% use Type 4.

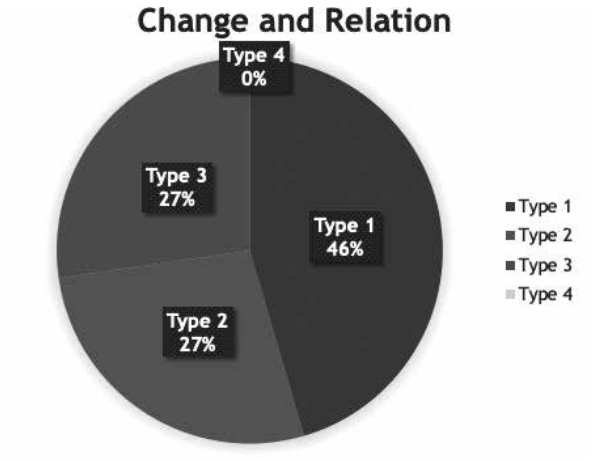


Fig. 12 Change and Relation

The figure 12 shows that 46% of the respondents use Type 1 of Neriage in teaching Change and Relation. Types 2 and 3 are used by 27%, and 0% use Type 4.

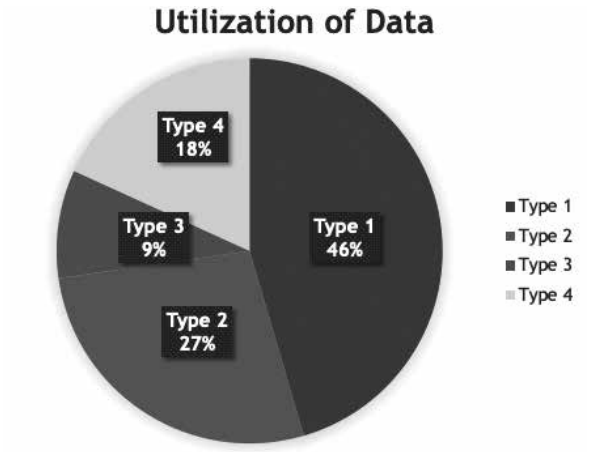


Fig. 13 Utilization of Data

The figure 13 shows that 46% of the respondents use Type 1 of *Neriage* in teaching Utilization of Data. Type 2 is used by 27%, 18% use Type 4, and 9% use Type 3.

4. How do math teachers facilitate the structured problem-solving approach?

After translating, reading, comparing, and grouping the responses of the teachers, two common problem-solving procedures were found. However, these two procedures are very similar.

Figure 14 shows a simpler lesson flow used by the teachers, which starts from presenting the problem of the day through giving time to students to figure the solution individually and/or by discussing it with their classmates. This is followed by *Neriage* then ends with generalization and application of skills to other problems/situations.

Figure 15 shows a slightly more complex

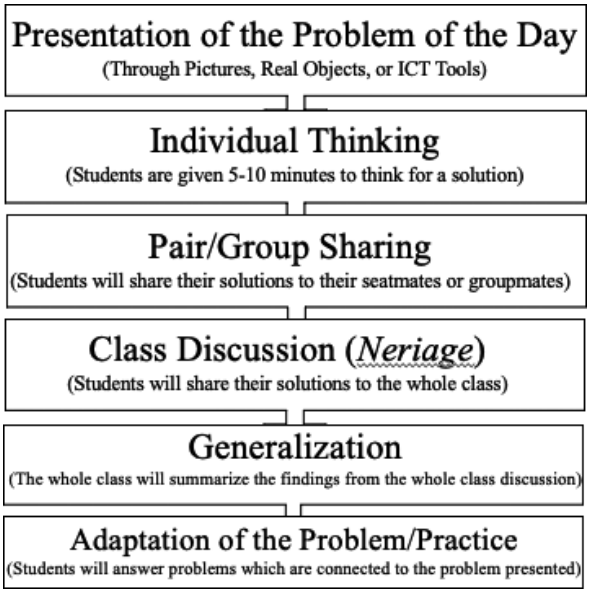


Fig. 14 Structured Problem Solving Approach 1

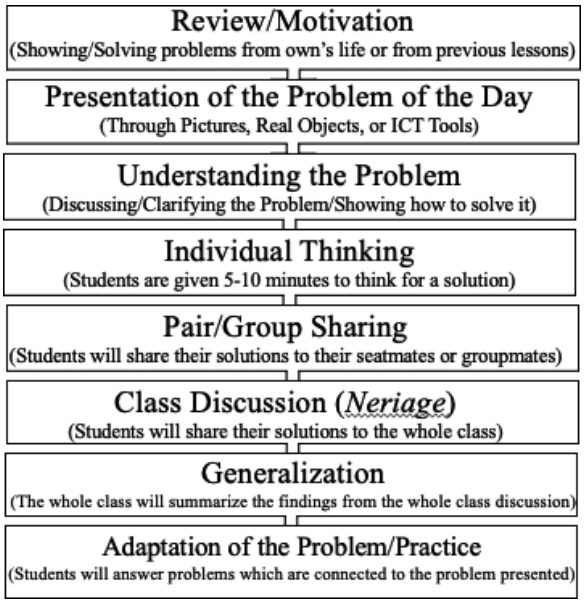


Fig. 15 Structured Problem Solving Approach 2

approach with the inclusion of the review of the past lesson and discussion of the problem that was presented.

5. How does *Neriage* support productive achievement of problem-solving tasks?

・Promoting Interactive Learning

Neriage encourages interactive learning by working through tasks and problems. When students interact with each other and discuss their thoughts/solutions, it makes their ideas and way of thinking better.

Communicating with other people, who have different ideas, makes the students learn. Through interaction, other techniques, which are easier to understand, can be learned.

・Improving/Extending Learning

Neriage supports multi-faceted learning. Knowing other ways to solve the problem improves students' way of thinking. By sharing and listening to other thoughts, students can learn various ways of thinking and multiple solutions to a single problem. Moreover, critical thinking can also be developed through comparing and differentiating students' ideas/solutions.

・Successful Achievement of Goals

Involving the students in the extensive discussion helps in successful achievement of goals. The proper organization of students' thought through *Neriage* and *Bansho* helps in coming up with an easy-to-understand solution. In addition, having deep discussion about confusion and solutions that can't be achieved by an individual student makes the achievement more accurate.

5. Consideration and Conclusion

This research followed 11 teachers who are teaching math in schools attached to Hokkaido University of Education. It is a case study that examines a certain group and place.

The results of the analysis reveal many commonalities among the instruments and methodologies, although some differences were seen from the *Neriage* approach used by the teachers during the lesson observations versus their answers in the questionnaire (Type 1 vs. Type 4).

Due to certain measures taken during the Covid-19 pandemic, a smaller group of respondents were observed and interviewed. A bigger and wider sample for future researches is being recommended.

The following conclusions have been drawn from the findings.

1. The mathematics textbooks used by the schools clearly support the structured problem-solving and *Neriage* approach, as they contain various word problems that are mostly open-ended.
2. The textbooks also cater to individual thinking as they contain activities that are based on the activities done in the classroom.
3. The common lesson flow that teachers follow supports the achievement of lesson objectives as well as the achievement of national standards.
4. The allotment of longer time to individual thinking and *Neriage* allows deeper discussion and production of possible solutions.
5. The use of textbooks as the one of the main sources of word problems for teachers suggests the alignment of lesson activities/goals with the textbooks' content.
6. The findings, in which most of the teachers opt to use pictures and ICT

tools in facilitating structured problem-solving, also support this lesson-to-textbook linkage as most of the pictures used by the teachers came from the textbook, and the ICT tools were from the digital version of the textbooks.

7. In terms of strategy in facilitating the *Neriage*, in general, the teachers commonly used the Type 1 (Developing a New Idea by Examining the Aggression of Thought) and Type 2 (Comparing Multiple Approaches to Learn a New Concept) approaches. However, some of the teachers also use different types for each strand, which suggests that the use of a certain type of *Neriage* is not limited to a specific strand/lesson.
8. Extensive discussion encourages interactive learning and communication skills as it involves group interaction.
9. Also, extensive discussion supports multi-faceted learning and critical thinking as it focuses individual and group thinking on a common problem. Through it, multiple ideas and solutions are being provided by the students.
10. Moreover, having deep discussions about the problem presented support successful achievement of goals since it involves the effort of the whole class.

From the lesson observations, textbook analyses, and questionnaire analysis, common traits and findings were found. Structured word problem-solving and the *Neriage* approach support the effective achievement of goals, production of multiple solutions to a problem, and development of higher-order thinking skills and mathematical skills among learners.

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