



拡大リー系に於けるリー微分について(II)

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On Lie Differentials in Extended Lie Systems, II.

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§ 1. Introduction.

This is a continuation of the previous paper of the same title [5]¹⁾. In the previous paper, we gave general formulas for Lie differentials relative to the basic frame and coframe and considered relationships between the Lie differentials and the covariant ones concerning the respective connections and finally, introduced an extended Lie subsystem. In this paper, first of all, we shall deal with general quantities and relations with respect to the basic frame and coframe. Next, we shall calculate the Lie derivatives of connections and give conditions for certain vector fields to be affine Killing vector fields. Finally, we shall also give conditions for projective Killing vector fields. The notations employed are the same as those of the previous papers [4] and [5].

§ 2. Quantities and relations with respect to the basic frame and coframe.

Let w_j^i and w_b^a be connection forms with respect to the natural and basic frames respectively. Then they are related by

$$(2. 1) \quad w_b^a = \xi_b^j w_j^i \xi_i^a + d\xi_b^i \xi_j^a.$$

Putting

$$(2. 2) \quad w_b^a = \Gamma_{bc}^a w^c, \quad w_j^i = \Gamma_{jk}^i dx^k,$$

from (2. 1) and (2. 2), we have

$$(2. 3) \quad \Gamma_{jk}^i = \Gamma_{ba}^i \xi_a^j \xi_j^b \xi_c^k + \xi_a^i \frac{\partial \xi_j^a}{\partial x^k}, \quad \text{or} \quad \Gamma_{ba}^a = \Gamma_{jk}^i \xi_a^j \xi_b^i \xi_c^k + \xi_j^a \xi_c^k \frac{\partial \xi_b^i}{\partial x^k}.$$

On making use of (2. 2) and $dw^a = -\frac{1}{2} \alpha_{bc}^a w^b \wedge w^c$, we calculate the curvature forms and get

$$\begin{aligned} \theta_b^a &= dw_b^a - w_b^e \wedge w_e^a \\ &= (\partial_c \Gamma_{ba}^a - \frac{1}{2} \Gamma_{bc}^a \alpha_{ca}^e - \Gamma_{ba}^e \Gamma_{ca}^a) w^c \wedge w^a \end{aligned}$$

1) Numbers in brackets refer to references at the end of the paper.

$$= \frac{1}{2}(\partial_c \Gamma_{ba}^a - \partial_a \Gamma_{bc}^a + \Gamma_{ba}^e \Gamma_{ec}^a - \Gamma_{bc}^e \Gamma_{ea}^a - a_{ca}^e \Gamma_{be}^a) w^c \wedge w^d.$$

From the above result, we have the components R_{bca}^a of the curvature tensor :

$$(2. 4) \quad R_{bca}^a = \partial_c \Gamma_{ba}^a - \partial_a \Gamma_{bc}^a + \Gamma_{ba}^e \Gamma_{ec}^a - \Gamma_{bc}^e \Gamma_{ea}^a - a_{ca}^e \Gamma_{be}^a.$$

For the components ζ_{bc}^a of the torsion tensor, we have

$$(2. 5) \quad \zeta_{bc}^a = -(a_{bc}^e + \Gamma_{bc}^a - I_{cb}^a)$$

In the previous papers [4] and [5], we gave the components of the curvature tensors and torsion tensors for the (+) and (0) connections. Here we give those with respect to the basic frames for the (-) connection.

For the curvature tensors, we have by virtue of (2. 4) and the expressions (2. 2) and (2. 5) in [4]

$$(2. 6) \quad \left\{ \begin{array}{l} \bar{R}_{bca}^a = 0, \\ \bar{R}_{bca}^{a*} = -\partial_b(a_{ca}^a - 2d_{[ca]}^a) + (a_{ca}^e - 2d_{[ca]}^e)d_{eb}^a - (a_{ed}^a - 2d_{[ea]}^a)d_{cb}^e \\ \quad + (a_{ec}^a - 2d_{[ca]}^a)d_{ab}^e, \\ \bar{R}_{bca}^{a(-)} = \frac{1}{4}\{-2\partial_b(a_{ca}^a - 2d_{[ca]}^a) + 2(a_{ca}^e - 2d_{[ca]}^e)d_{eb}^a - (a_{ed}^e - 2d_{[ea]}^e) \\ \quad (a_{ec}^a - 2d_{[ca]}^a) + (a_{bc}^e - 2d_{[bc]}^e)(a_{ca}^a - 2d_{[ca]}^a)\}. \end{array} \right.$$

For the torsion tensors, we have

$$(2. 7) \quad \bar{\zeta}_{bc}^a = -a_{bc}^a + 2d_{[bc]}^a, \quad \bar{\zeta}_{bc}^{a*} = a_{bc}^a - 2d_{[bc]}^a, \quad \bar{\zeta}_{bc}^{a(-)} = 0.$$

Let T be a tensor field of type (1, 1) and let $\nabla_k T_j^i$ and $\nabla_c T_b^a$ be the covariant derivatives with respect to the natural and basic frames respectively. Then, since

$$\nabla_k T_j^i \frac{\partial}{\partial x^i} \otimes dx^j \otimes dx^k = \nabla_c T_b^a X_a \otimes w^b \otimes w^c,$$

it follows that

$$(2. 8) \quad \nabla_c T_b^a = \partial_c T_b^a + \Gamma_{ac}^a T_b^a - \Gamma_{bc}^a T_a^a.$$

Thus we see that the covariant differentiation relative to the basic frames is formally the same as that relative to the natural ones and so for tensors of the general type.

Since $[X_a, X_b] = a_{ab}^e X_e$, it follows that

$$(2. 9) \quad \partial_a \partial_b T - \partial_b \partial_a T = a_{ab}^e \partial_e T,$$

where T is any quantity relative to the basic frames.

From (2. 4), (2. 5), (2. 8), and (2. 9), we have

$$(2. 10) \quad \nabla_a \nabla_c T_b^a - \nabla_c \nabla_a T_b^a = -T_b^e R_{eca}^a + T_e^a R_{bca}^e + \nabla_e T_b^a \Omega_{ca}^e,$$

as relative to the natural frames.

§ 3. Affine Killing vector fields.

As is well known, a necessary and sufficient condition for a vector field $V = v^i \frac{\partial}{\partial x^i}$

to be an affine Killing vector one is given by

$$(3. 1) \quad \mathcal{L}_v \Gamma_{jk}^i = \nabla_k \{ V_j v^i + (\Gamma_{jl}^i - \Gamma_{lj}^i) v^l \} - R_{ikl}^i v^l = 0,$$

where $\mathcal{L}_v \Gamma_{jk}^i$ is the Lie derivative of a connection Γ with respect to the vector field V .

And $\mathcal{L}_v \Gamma_{jk}^i$ can be also expressed in

$$(3. 2) \quad \mathcal{L}_v \Gamma_{jk}^i = \frac{\partial^2 v^i}{\partial x^j \partial x^k} + v^l \frac{\partial \Gamma_{jk}^i}{\partial x^l} + \Gamma_{jl}^i \frac{\partial v^l}{\partial x^k} + \Gamma_{lk}^i \frac{\partial v^l}{\partial x^j} - \Gamma_{jk}^l \frac{\partial v^i}{\partial x^l}.$$

Putting

$$\mathcal{L}_v \Gamma_{jk}^i \frac{\partial}{\partial x^i} \otimes dx^j \otimes dx^k = \mathcal{L}_v \Gamma_{ba}^a X_a \otimes w^b \otimes w^c,$$

we have

$$(3. 3) \quad \mathcal{L}_v \Gamma_{ba}^a = \nabla_c (\nabla_b v^a - \mathcal{L}_{ba}^a v^a) - R_{bac}^a v^a,$$

where $\mathcal{L}_{ba}^a v^a$ and v^a are components relative to the basic frames corresponding to $\mathcal{L}_v \Gamma_{jk}^i$ and v^i respectively.

On making use of (2. 4), (2. 5), (3. 3) and the covariant differentiation relative to the basic frames, we have

$$(3. 4) \quad \begin{aligned} \mathcal{L}_v \Gamma_{ba}^a &= \nabla_c \{ \partial_b v^a + (a_{ba}^a + \Gamma_{ba}^a) v^a \} - R_{bac}^a v^a \\ &= \partial_c \partial_b v^a + (a_{ba}^a + \Gamma_{ba}^a) \partial_c v^a + \Gamma_{ac}^a \partial_b v^a - \Gamma_{bc}^c \partial_c v^a \\ &\quad + (\partial_c a_{ba}^a + \partial_a \Gamma_{bc}^a + a_{ba}^c \Gamma_{ec}^a - \Gamma_{bc}^e a_{ea}^a + a_{cd}^e \Gamma_{be}^a) v^d, \end{aligned}$$

which can be also obtained by substituting (2. 3) in (3. 2).

For the respective connections, we have

$$(3. 5) \quad \begin{aligned} \overset{+}{\Gamma}_{bc}^a &= 0, & \overset{+}{\Gamma}_{bc}^* &= -a_{bc}^a, & \overset{+}{\Gamma}_{bc}^{(+)} &= -\frac{1}{2} a_{cb}^a \\ \overset{-}{\Gamma}_{bc}^a &= -d_{bc}^a, & \overset{-}{\Gamma}_{bc}^* &= a_{cb}^a - d_{cb}^a, & \overset{-}{\Gamma}_{bc}^{(-)} &= -\frac{1}{2} (a_{bc}^a + 2d_{bc}^a) \\ \overset{0}{\Gamma}_{bc}^a &= -\frac{1}{2} d_{bc}^a, & \overset{0}{\Gamma}_{bc}^* &= a_{cb}^a - \frac{1}{2} d_{cb}^a, & \overset{0}{\Gamma}_{bc}^{(0)} &= -\frac{1}{2} (a_{bc}^a + d_{bc}^a), \end{aligned}$$

By virtue of (2. 9), (3. 3), (3. 4) and (3. 5), we have the following :

$$(3. 6) \quad \left\{ \begin{aligned} \mathcal{L}_v \overset{+}{\Gamma}_{bc}^a &= \overset{+++}{\nabla_c} \nabla_b v^a, \quad \mathcal{L}_v \overset{+}{\Gamma}_{bc}^* = \overset{+++}{\nabla_b} \nabla_c v^a = \overset{++}{\nabla_c} \nabla_b v^a - \overset{+}{R}_{bac}^* v^a \\ \mathcal{L}_v \overset{+}{\Gamma}_{bc}^{(+)} &= \frac{1}{2} (\overset{+++}{\nabla_c} \nabla_b v^a + \overset{+++}{\nabla_b} \nabla_c v^a) = \overset{+(+)}{\nabla_c} \nabla_b v^a - \overset{(+)}{R}_{bac}^{(+)} v^a, \end{aligned} \right.$$

$$(3. 7) \quad \left\{ \begin{aligned} \mathcal{L}_v \overset{-}{\Gamma}_{bc}^a &= \overset{---}{\nabla_c} \nabla_b v^a, \quad \mathcal{L}_v \overset{-}{\Gamma}_{bc}^* = \overset{---}{\nabla_b} \nabla_c v^a = \overset{--}{\nabla_c} \nabla_b v^a - \overset{-}{R}_{bac}^* v^a \\ \mathcal{L}_v \overset{-}{\Gamma}_{bc}^{(-)} &= \frac{1}{2} (\overset{---}{\nabla_c} \nabla_b v^a + \overset{---}{\nabla_b} \nabla_c v^a) = \overset{(-)}{\nabla_c} \nabla_b v^a - \overset{(-)}{R}_{bac}^{(-)} v^a \end{aligned} \right.$$

$$(3. 8) \quad \left\{ \begin{aligned} \mathcal{L}_v \overset{0}{\Gamma}_{bc}^a &= \overset{0--}{\nabla_c} \nabla_b v^a + \frac{1}{2} \overset{+}{\nabla_c} (d_{ba}^a v^a) = \overset{00*}{\nabla_c} \nabla_b v^a - \overset{0}{R}_{bac}^a v^a \\ \mathcal{L}_v \overset{0*}{\Gamma}_{bc}^a &= \overset{0--}{\nabla_b} \nabla_c v^a + \frac{1}{2} \overset{+}{\nabla_b} (d_{ca}^a v^a) = \overset{0*0}{\nabla_c} \nabla_b v^a - \overset{0*}{R}_{bac}^a v^a \\ \mathcal{L}_v \overset{(0)}{\Gamma}_{bc}^a &= \frac{1}{2} [\overset{0--}{\nabla_c} \nabla_b v^a + \overset{0--}{\nabla_b} \nabla_c v^a + \frac{1}{2} \{ \overset{+}{\nabla_c} (d_{ba}^a v^a) + \overset{+}{\nabla_b} (d_{ca}^a v^a) \}] = \overset{(0)(0)}{\nabla_c} \nabla_b v^a - \overset{(0)}{R}_{bac}^a v^a. \end{aligned} \right.$$

From (3. 6) it follows that if $\overset{+*}{\nabla}_b v^a = 0$, $\mathcal{L}_v \overset{+}{\Gamma}_{ba}^a = \mathcal{L}_v \overset{+*}{\Gamma}_{ba}^a = \mathcal{L}_v \overset{(+)}{\Gamma}_{ba}^a = 0$. Hence we get the following theorem :

Theorem 1. *If a vector field V is parallel for the $(+*)$ connection, the field V is an affine Killing vector one for any of the $(+)$, $(+*)$ and $((+))$ connections.*

$$\text{If } \overset{-*}{\nabla}_b v^a = \partial_b v^a + (a_{ba}^a - d_{ba}^a)v^d = 0, \text{ it implies}$$

$$\overset{+}{\nabla}_c (d_{ba}^a v^d) = (\partial_c d_{ba}^a + d_{ca}^e d_{be}^a - a_{ca}^e d_{be}^a)v^d.$$

From (3. 7), (3. 8) and the above result, we have :

Theorem 2. *If a vector field $V=(v^a)$ is parallel for the $(-*)$ connection, the field V is an affine Killing vector one for any of the $(-)$, $(-*)$ and $((-))$ connections. If, furthermore, the components v^a satisfy the following equations :*

$$(\partial_c d_{ba}^a + d_{ca}^e d_{be}^a - a_{ca}^e d_{be}^a)v^d = 0,$$

then the field V is also an affine Killing vector one for any of the (0) , (0) and $((0))$ connections.*

From (3. 6), (3. 7) and (3. 8), we obtain :

Theorem 3. *Necessary and sufficient conditions for a vector field $V=(v^a)$ to be an affine Killing vector one for the respective connections are the following :*

$$\left\{ \begin{array}{l} \text{For either of the } (+) \text{ and } (+*) \text{ conn., } \partial_b v^a + a_{ba}^a v^d = \text{const.} \\ \text{For the } ((+)) \text{ conn., } \partial_c (\partial_b v^a + a_{ba}^a v^d) + \partial_b (\partial_c v^a + a_{ca}^a v^d) = 0. \end{array} \right.$$

$$\left\{ \begin{array}{l} \text{For either of the } (-) \text{ and } (-*) \text{ conn., } \overset{-}{\nabla}_i \overset{-}{\nabla}_c v^a = 0. \\ \text{For the } ((-)) \text{ conn., } \overset{-}{\nabla}_i \overset{-}{\nabla}_c v^a + \overset{-}{\nabla}_c \overset{-}{\nabla}_i v^a = 0. \end{array} \right.$$

$$\left\{ \begin{array}{l} \text{For either of the } (0) \text{ and } (0*) \text{ conn., } \overset{0}{\nabla}_i \overset{0}{\nabla}_c v^a + \frac{1}{2} \partial_b (d_{ca}^a v^d) = 0. \\ \text{For the } ((0)) \text{ conn., } 2 (\overset{0}{\nabla}_c \overset{0}{\nabla}_b v^a + \overset{0}{\nabla}_b \overset{0}{\nabla}_c v^a) + \partial_c (d_{ba}^a v^d) + \partial_b (d_{ca}^a v^d) = 0. \end{array} \right.$$

Forthwith we have :

Corollary 3. 1. *If a vector field V is an affine Killing vector one for either of the $(+)$ and $(+*)$ connections, so also is the field for the $((+))$ connection. And similarly for the $(-)$ and (0) connections.*

If in particular, $V=(\delta_{\alpha}^{\alpha})$ (α ; fixed), we have :

Corollary 3. 2. *Necessary and sufficient conditions for the vector field X_{α} , (α ; fixed) to be an affine Killing vector one for the respective connections are the following :*

$$\left\{ \begin{array}{l} \text{For either of the } (+) \text{ and } (+*) \text{ conn., } a_{ba}^a = \text{const.} \\ \text{For the } ((+)) \text{ conn., } \partial_c a_{b\alpha}^a + \partial_b a_{c\alpha}^a = 0. \end{array} \right.$$

$$\left\{ \begin{array}{l} \text{For either of the } (-) \text{ and } (-*) \text{ conn., } \partial_b b_{ca}^a - b_{ba}^a d_{cc}^a + d_{bc}^a b_{ca}^a = 0 \quad (b_{ca}^a = a_{ca}^a - d_{ca}^a). \\ \text{For the } ((-)) \text{ conn., } \partial_b b_{ca}^a + \partial_c b_{ba}^a - (b_{ba}^a d_{cc}^a + b_{ca}^a d_{cb}^a) + 2d_{(bc)}^a b_{ca}^a = 0, \\ \text{For either of the } (0) \text{ and } (0*) \text{ conn., } \partial_b (a_{ca}^a + b_{ca}^a) + d_{cb}^a b_{ca}^a - b_{ca}^a d_{ab}^a = 0. \\ \text{For the } ((0)) \text{ conn., } \partial_b (a_{ca}^a + b_{ca}^a) + \partial_c (a_{ba}^a + b_{ba}^a) + 2d_{(bc)}^a b_{ca}^a - (b_{ca}^a d_{cb}^a + b_{ba}^a d_{cc}^a) = 0. \end{array} \right.$$

§ 4. Projective Killing Vector fields.

The ((+)) connection is the symmetric connection whose coefficients are the symmetric part of those of the (+) and (+*) connections, and so the ((-)) and ((0)) connections for the (-) & (-*) and (0) & (0*) connections respectively. If the functions d_{bc}^a are skewsymmetric in the indices a and b , the ((+)), ((-)) and ((0)) connections identify with one another by the theorem 4 in [4], and consequently the paths are the same for all the nine connections, that is, all the connections are one another in projective correspondence. Now, we seek for another condition admitting the above statement.

A necessary and sufficient condition that two connections $\overset{(+)}{\Gamma}$ and $\overset{(-)}{\Gamma}$ be in projective correspondence is that there exists a covariant vector field (ϕ_k) satisfying the following relation :

$$(4. 1) \quad \overset{(+)}{\Gamma}_{jk}^i = \overset{(-)}{\Gamma}_{jk}^i + \phi_j \delta_k^i + \phi_k \delta_j^i.$$

Since $\overset{(-)}{\Gamma}_{ik}^i = \overset{(+)}{\Gamma}_{jk}^i - d_{(bc)}^a \xi_a^i \xi_j^b \xi_k^c$ [5], it follows from (4. 1) that

$$d_{(bc)}^a \xi_a^i \xi_j^b \xi_k^c = \phi_j \delta_k^i + \phi_k \delta_j^i,$$

from which we have

$$(4. 2) \quad d_{(bc)}^a = \phi_j (\xi_b^i \delta_c^a + \xi_c^j \delta_b^a).$$

On the other hand, for the connection $\overset{(0)}{\Gamma}$ we have

$$(4. 3) \quad \overset{(0)}{\Gamma}_{jk}^i = \overset{(+)}{\Gamma}_{jk}^i - \frac{1}{2} d_{(bc)}^a \xi_a^i \xi_j^b \xi_k^c \quad [4] = \overset{(+)}{\Gamma}_{jk}^i - \frac{1}{2} (\phi_j \delta_k^i + \phi_k \delta_j^i).$$

By virtue of (4. 2) and (4. 3) we obtain :

Theorem 4. *A necessary and sufficient condition that all the nine connections be mutually in projective correspondence is that the functions d_{bc}^a satisfy the following equations :*

$$\begin{array}{l} 1) \quad d_{bc}^a + d_{cb}^a = 0 \quad (a \neq b, a \neq c) \\ 2) \quad d_{cc}^a = d_{ac}^a + d_{ca}^a \quad (a \neq c, a, c ; \text{ not summed}), \end{array}$$

provided that all of d_{cc}^c do not vanish.

For the above theorem, the components ϕ_j in (4. 1) (or in (4. 3)) are

$$\phi_j = \frac{1}{2} \sum_a d_{aa}^a \xi_j^a = \frac{1}{n+1} \sum_{a,c} d_{(ca)}^c \xi_j^a,$$

Next, we seek for conditions for a vector field $V=(v^a)$ to be a projective Killing vector one. We know that a necessary and sufficient condition for the field V to be

a projective Killing vector one for a condition Γ is that there exists a covariant vector field (ϕ_a) satisfying

$$(4. 4) \quad \mathbb{L}_v \Gamma_{bc}^a = \phi_b \delta_c^a + \phi_c \delta_b^a.$$

On making use of (3. 6), (3. 7), (3. 8) and (4. 4), we have :

Theorem 5. *Necessary and sufficient conditions for a vector field $V=(v^a)$ to be a projective Killing vector one for each of the $((+))$, $((-))$ and $((0))$ connections are that the components v^a satisfy the following relations :*

$$\text{For the } ((+)) \text{ conn., } \partial_c(\nabla_b^+ v^a) + \partial_b(\nabla_c^+ v^a) = 0 \quad (a \neq b, a \neq c),$$

$$\partial_c(\nabla_c^+ v^a) = \partial_a(\nabla_c^+ v^a) + \partial_c(\nabla_a^+ v^a) \quad (a \neq c, a, c ; \text{ not summed}),$$

provided that all of $\partial_c \nabla_c^+ v^a$ do not vanish.

$$\text{For the } ((-)) \text{ conn., } \nabla_c \nabla_b^- v^a + \nabla_b \nabla_c^- v^a = 0 \quad (a \neq b, a \neq c),$$

$$\nabla_c \nabla_c^- v^a = \nabla_a \nabla_c^- v^a + \nabla_c \nabla_a^- v^a \quad (a \neq c, a, c ; \text{ not summed}),$$

provided that all of $\nabla_c \nabla_c^- v^a$ do not vanish.

$$\text{For the } ((0)) \text{ conn., } \nabla_a \nabla_b^0 v^a + \nabla_b \nabla_a^0 v^a + \frac{1}{2} \{ \partial_c(d_{ba}^a v^a) + \partial_b(d_{ca}^a v^a) \} = 0 \quad (a \neq b, a \neq c)$$

$$\nabla_c \nabla_c^0 v^a + \frac{1}{2} \partial_c(d_{ca}^a v^a) = \nabla_a \nabla_c^0 v^a + \nabla_c \nabla_a^0 v^a + \frac{1}{2} \{ \partial_c(d_{aa}^a v^a) + \partial_a(d_{ca}^a v^a) \},$$

$$(a \neq c, a, c ; \text{ not summed})$$

provided that all of the expressions $\nabla_c \nabla_c^0 v^a + \frac{1}{2} \partial_c(d_{ca}^a v^a)$ do not vanish.

From the above theorem, we have :

Corollary 5. 1. *Necessary and sufficient conditions for the vector field X_α ($\alpha ;$ fixed) to be a projective Killing vector one for each of the $((+))$, $((-))$ and $((0))$ connections are the following :*

$$\text{For the } ((+)) \text{ conn., } \partial_c a_{b\alpha}^a + \partial_b a_{c\alpha}^a = 0 \quad (a \neq b, a \neq c)$$

$$\partial_c a_{c\alpha}^a = \partial_a a_{c\alpha}^a + \partial_c a_{a\alpha}^a \quad (a \neq c, a, c ; \text{ not summed}),$$

provided that all of $a_{c\alpha}^a$ are not constants.

$$\text{For the } ((-)) \text{ conn., } \partial_c b_{b\alpha}^a + \partial_b b_{c\alpha}^a - (b_{c\alpha}^e d_{eb}^a + b_{b\alpha}^e d_{ec}^a) + 2d_{(bc)}^e b_{e\alpha}^a = 0 \quad (a \neq b, a \neq c),$$

$$\partial_c (b_{c\alpha}^a - b_{a\alpha}^c) - \partial_a b_{c\alpha}^a + (d_{ca}^a - d_{cc}^a) b_{c\alpha}^a + d_{cc}^e b_{e\alpha}^a$$

$$- 2d_{(ca)}^e b_{e\alpha}^a + b_{a\alpha}^e d_{ec}^a = 0 \quad (a \neq c, a, c ; \text{ not summed}).$$

provided that all of the expressions $\partial_c b_{c\alpha}^a - b_{c\alpha}^e d_{ec}^a + d_{cc}^e b_{e\alpha}^a$ do not vanish.

$$\text{For the } ((0)) \text{ conn., } \partial_b (a_{b\alpha}^a + b_{c\alpha}^a) + \partial_c (a_{b\alpha}^a + b_{b\alpha}^c) + 2d_{(bc)}^e b_{e\alpha}^a - (b_{c\alpha}^e d_{eb}^a + b_{b\alpha}^e d_{ec}^a) = 0$$

$$(a \neq b, a \neq c)$$

$$\partial_c (a_{c\alpha}^a + b_{c\alpha}^c - a_{a\alpha}^a - b_{a\alpha}^c) - \partial_a (a_{c\alpha}^a + b_{c\alpha}^c) - b_{c\alpha}^e (d_{cc}^a - d_{ea}^a)$$

$$+ d_{cc}^e b_{e\alpha}^c - 2d_{(ca)}^e b_{e\alpha}^a + b_{a\alpha}^e d_{ec}^a = 0 \quad (a \neq c, a, b ; \text{ not summed})$$

provided that all of the expressions $\partial_c (a_{c\alpha}^a + b_{c\alpha}^c) - d_{cc}^e b_{e\alpha}^c + d_{cc}^e b_{e\alpha}^c$ do not vanish.

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