Remark on the ρ-conformally Flat Riemannian Manifold

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Remark on the p-conformally Flat Riemannian Manifold

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長谷川和泉：p-共形平坦なリーマン空間について

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Abstract

Let M be an n-dimensional Riemannian manifold. As the generalization of the conformal curvature tensor, R. S. Kulkarni [2] defined the p-th confomal curvature tensor. The Riemannian manifold M is called the p-conformally flat one if the p-th conformal curvature tensor vanishes and $n \geqq 4p$.

In the previous paper[1], we gave one characterization of the p-conformally flat Riemannian manifold. In this paper, we shall give another characterization of the p-conformally flat Riemannian manifold.

1. Preliminaries

For terminology and notation, we follow the previous paper[1]. The p-th conformal curvature tensor C_p is given by

(1. 1) $$C_p = R^p + \sum_{k=1}^{2p} \frac{(-1)^k}{k!\prod_{j=0}^{k-1}(n-4p+2+j)} g^k \wedge c^k R^p.$$

Of course, C_1 is the Weyl conformal curvature tensor.

T. Nasu and M. Kojima[3] proved the following

Lemma. *A Riemannian manifold M of dimension* $n(\geqq 4p)$ *is p-conformally flat if and only if* R^p *satisfies*

(1. 2) $$\sum_{\alpha} \varepsilon(\alpha) R^p(X_{\alpha_1}, X_{\alpha_2}, \cdots, X_{\alpha_{2p}})(X_{\alpha_1}, X_{\alpha_2}, \cdots, X_{\alpha_{2p}}) = 0$$

for every $4p$*-tuple of orthonormal vectors* $\{X_1, X_2, \cdots, X_{4p}\}$ *at each point of M.*

Here, the sum extends over all $2p$-tuples $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_{2p})$ with $\alpha_i = i$ or $i+2p$ and $\varepsilon(\alpha) = 1$

or -1 according to the number of α_i such that $\alpha_i = i+2p$ is even or odd.

2. Theorem

Theorem. *In order that a Riemannian manifold M of dimension* $n\,(\geqq 4p)$ *is p-conformally flat, it is necessary and sufficient that* R^p *satisfies*

$$R^p(X_1, X_2, \cdots, X_{2p})(X_{2p+1}, X_{2p+2}, \cdots, X_{4p}) = 0 \tag{2. 1}$$

for every $4p$*-tuple of orthonormal vectors* $\{X_1, X_2, \cdots, X_{4p}\}$ *at each point of M.*

Proof. If M is p-conformally flat and $\{X_1, X_2, \cdots, X_{4p}\}$ is any $4p$-tuple of orthonormal vectors, from (1. 1), it is clear that R^p satisfies (2. 1).

Conversely, we assume that R^p satisfies (2. 1) for every $4p$-tuple of orthonormal vectors $\{X_1, X_2, \cdots, X_{4p}\}$.

It is clear that $\{X_1+X_{2p+1}, X_2+X_{2p+2}, \cdots, X_{2p}+X_{4p}, X_1-X_{2p+1}, X_2-X_{2p+2}, \cdots, X_{2p}-X_{4p}\}$ is the $4p$-tuple of orthogonal vectors.

From this fact and simple computations, we have (1. 2). Using the lemma, we conclude the theorem. Q. E. D.

References

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