A Note on Hypersurfaces in a Sasakian Space Form

北海道教育大学紀要（第２部Ａ）第41巻　第１号
Journal of Hokkaido University of Education (Section II A) Vol. 41, No. 1

平成２年９月
September, 1990

A Note on Hypersurfaces in a Sasakian Space Form

Izumi HASEGAWA
Mathematics Laboratory, Sapporo College,
Hokkaido University of Education
Sapporo 002

佐々木空間形の超曲面についての一注意

長谷川　和　泉

北海道教育大学札幌分校数学教室

Abstract

This is a study of hypersurfaces in a Sasakian space form $\tilde{M}(c)$ of dimension $2m+1(\geq 5)$ and provides proof that there are no Einstein hypersurfaces in $\tilde{M}(c)$, $c \neq 1$.

§ 1. Introduction.

It is well known that there are no Einstein real hypersurfaces in a complex space form $M(c)$ with nonzero constant holomorphic sectional curvature c ([4] and [5]). Recently U-Hang Ki[3] found that there are no real hypersurfaces with parallel Ricci tensor in a complex space form $M(c)$, $c \neq 0$.

In this note we investigate the hypersurfaces in a Sasakian space form of dimension $2m+1$ (≥ 5) and prove the following:

THEOREM. *There are no Einstein hypersurfaces in a Sasakian space form with constant φ-holomorphic sectional curvature $c(\neq 1)$.*

Throughout this paper, we assume that all objects under consideration are differentiable of class C^∞.

§ 2. Preliminaries.

This section introduces definitions and basic properties which are used in the following

sections.

(1) Let M be a $(2m+1)$-dimensional Sasakian manifold, and $(\varphi, \xi, \eta, \langle\ ,\ \rangle)$ a Sasakian structure of M where φ is a tensor field of type (1, 1), ξ a vector field, η a 1-form and $\langle\ ,\ \rangle$ a Riemannian metric. Then the structure tensors satisfy the following equations:

$$\begin{aligned}&\varphi^2 X=-X+\eta(X)\xi,\quad \varphi\xi=0,\quad \eta(\varphi X)=0,\quad \eta(\xi)=1,\\&\langle\varphi X, Y\rangle+\langle X, \varphi Y\rangle=0,\quad \eta(X)=\langle\xi, X\rangle,\\&\nabla_X\xi=\varphi X,\quad (\nabla_X\varphi)Y=\eta(Y)X-\langle X, Y\rangle\xi\end{aligned}\tag{2.1}$$

for any vector fields X, Y tangent to M, where ∇ denotes the Riemannian connection of M.

The Riemannian curvature tensor R of the Sasakian manifold M, defined by $R(X, Y)Z=\nabla_X\nabla_Y Z-\nabla_Y\nabla_X Z-\nabla_{[X,Y]}Z$, satisfies

$$\begin{aligned}&R(\varphi X, \varphi Y)Z=R(X, Y)Z-\langle Y, Z\rangle X+\langle X, Z\rangle Y-\langle Y, \varphi Z\rangle\varphi X+\langle X, \varphi Z\rangle\varphi Y,\\&R(X, \varphi Y)Z=-R(\varphi X, Y)Z-\langle Y, \varphi Z\rangle X+\langle X, \varphi Z\rangle Y+\langle Y, Z\rangle\varphi X-\langle X, Z\rangle\varphi Y\\&\text{and}\\&R(X, Y)\xi=\eta(Y)X-\eta(X)Y.\end{aligned}\tag{2.2}$$

A Sasakian manifold M is called a Sasakian space form if M is of constant φ-holomorphic sectional curvature. The Riemannian curvature tensor R of a Sasakian space form $M(c)$ of constant φ-holomorphic sectional curvature c takes the following form:

$$\begin{aligned}R(X, Y)Z=&\frac{c+3}{4}\{\langle Y, Z\rangle X-\langle X, Z\rangle Y\}+\frac{c-1}{4}\{\eta(X)\eta(Z)Y-\eta(Y)\eta(Z)X\\&+\langle X, Z\rangle\eta(Y)\xi-\langle Y, Z\rangle\eta(X)\xi+\langle X, \varphi Z\rangle\varphi Y-\langle Y, \varphi Z\rangle\varphi X\\&+2\langle X, \varphi Y\rangle\varphi Z\}.\end{aligned}\tag{2.3}$$

(2) Let $\tilde{M}$ be a Riemannian manifold and M a Riemannian manifold isometrically immersed in $\tilde{M}$. Then M is called a submanifold in $\tilde{M}$, and when codim $M=1$ it is called a hypersurface in $\tilde{M}$. We denote the Riemannian metric on M as well as the induced metric on M by $\langle\ ,\ \rangle$. Let ∇ and $\tilde{\nabla}$ be the Riemannian connections on M and $\tilde{M}$, respectively. Then the Gauss and Weingarten formulas are given by

$$\tilde{\nabla}_X Y=\nabla_X Y+\sigma(X, Y)\tag{2.4}$$

and

$$\tilde{\nabla}_X N=A_N X+\nabla^{\perp}_X N\tag{2.5}$$

for any vector fields X, Y tangent to M and N normal to M, where σ denotes the second fundamental form, A_N the second fundamental tensor at N, and $\nabla^{\perp}$ the linear connection induced in the normal bundle $T^{\perp}M$, called the normal connection. The second fundamental tensor A_N is related to the second fundamental form σ by

(2. 6) $\langle A_N X, Y\rangle=\langle\sigma(X, Y), N\rangle$.

When denoting the Riemannian curvature tensors of M and $\tilde{M}$ by R and $\tilde{R}$, the equations of Gauss and Codazzi are given by

(2. 7) $\langle W, R(X, Y)Z\rangle=\langle W, \tilde{R}(X, Y)Z\rangle+\langle\sigma(W, X), \sigma(Y, Z)\rangle-\langle\sigma(W, Y), \sigma(X, Z)\rangle$

and

(2. 8) $\langle\tilde{R}(X, Y)Z, N\rangle=\langle(\bar{\nabla}_X\sigma)(Y, Z), N\rangle-\langle(\bar{\nabla}_Y\sigma)(X, Z), N\rangle$

for any vectors W, X, Y, Z tangent to M and N normal to M, where the first covariant differentiation $\bar{\nabla}\sigma$ of σ is defined by

(2. 9) $(\bar{\nabla}_X\sigma)(Y, Z)=\nabla^{\perp}_X\sigma(Y, Z)-\sigma(\nabla_X Y, Z)-\sigma(Y, \nabla_X Z)$.

A submanifold M is called a parallel submanifold if σ is parallel, i. e., $\bar{\nabla}\sigma=0$ identically.

(3) Let M be a hypersurface in a $(2m+1)$-dimensional Sasakian manifold $\tilde{M}$ with Sasakian structure $(\tilde{\varphi}, \tilde{\xi}, \tilde{\eta}, \langle\ ,\ \rangle)$. A unit normal ε to M may then be chosen. For this unit normal ε, we put

(2. 10) $$\begin{aligned} &f:=\tilde{\eta}(\varepsilon),\quad \zeta:=-\tilde{\varphi}\varepsilon,\quad \xi:=\tilde{\xi}-f\varepsilon,\quad \varphi X:=\tilde{\varphi}X-\langle\zeta, X\rangle\varepsilon,\\ &AX:=A_\varepsilon X\quad\text{and}\quad h(X, Y):=\langle AX, Y\rangle=\langle\sigma(X, Y), \varepsilon\rangle\end{aligned}$$

for any vectors X, Y tangent to M. By the properties of the Sasakian structure, the following relations are given:

(2. 11) $$\begin{aligned} &\langle\zeta, \xi\rangle=0,\quad \|\zeta\|^2=\|\xi\|^2=1-f^2,\quad \varphi\zeta=-f\xi,\quad \varphi\xi=f\zeta,\\ &\varphi^2 X=-X+\langle\zeta, X\rangle\zeta+\eta(X)\xi,\quad \langle\varphi X, Y\rangle=-\langle X, \varphi Y\rangle,\\ &\nabla_X\zeta=\varphi AX-fX,\quad \nabla_X\xi=\varphi X+fAX,\quad Xf=\langle\zeta-A\xi, X\rangle,\\ &(\nabla_X\varphi)Y=\langle\zeta, Y\rangle AX-\langle AX, Y\rangle\zeta+\eta(Y)X-\langle X, Y\rangle\xi\end{aligned}$$

for any vectors X, Y tangent to M, where $\eta(X):=\langle\xi, X\rangle$(e. g., see [8]).

A scalar function $\rho:=\dfrac{1}{2m}\operatorname{trace} A$ is called the mean curvature of M in $\tilde{M}$. M is said to be totally umbilical, if $AX=\rho X$ for any vector X tangent to M, and M is said to be totally geodesic, if $AX=0$ for any vector X tangent to M.

We put $M_o:=\{x\in M\mid(1-f^2)(x)\neq 0\}$. Assuming that $\dim\tilde{M}\geq 5$, then M_o is an open dense set in M. In fact, if there exists some point x satisfying $\xi=\tilde{\xi}$ on a neighborhood of x, the Weingarten formula (2. 5) implies

$$\langle\varphi X, Y\rangle=\langle\tilde{\varphi}X, Y\rangle=\langle\tilde{\nabla}_X\tilde{\xi}, Y\rangle=-\langle AX, Y\rangle$$

for any tangent vector fields X and Y on the neighborhood of x. Therefore M is anti-invariant at x, i. e., $\tilde{\varphi}(T_x M)\subset T^{\perp}_x M$ and $\dim M\leq m$ (e. g., see Proposition 1.1 in Chapter III of [10]). Since M is the hypersurface in $\tilde{M}$, this is a contradiction.

In the following, the ambient Sasakian manifold is assumed to be a Sasakian space form

$\tilde{M}(c)$ of dimension $2m+1$. Then the equations of Gauss and Codazzi for M in $\tilde{M}(c)$ are rewritten as:

$$R(X, Y)Z=\frac{c+3}{4}(\langle Y, Z\rangle X-\langle X, Z\rangle Y)+\frac{c-1}{4}\{\eta(X)\eta(Z)Y-\eta(Y)\eta(Z)X$$

(2.12)
$$+\langle X, Z\rangle\eta(Y)\xi-\langle Y, Z\rangle\eta(X)\xi+\langle X, \varphi Z\rangle\varphi Y-\langle Y, \varphi Z\rangle\varphi X$$
$$+2\langle X, \varphi Y\rangle\varphi Z\}+\langle AY, Z\rangle AX-\langle AX, Z\rangle AY,$$

$$(\nabla_X A)Y-(\nabla_Y A)X=\frac{c-1}{4}\{f(\eta(Y)X-\eta(X)Y)+\langle\xi, X\rangle\varphi Y-\langle\xi, Y\rangle\varphi X$$

(2.13)
$$+2\langle X, \varphi Y\rangle\xi\},$$

i. e.,

$$(\nabla_X h)(Y, Z)-(\nabla_Y h)(X, Z)=\frac{c-1}{4}\{f(\eta(Y)\langle X, Z\rangle-\eta(X)\langle Y, Z\rangle)$$
$$+\langle\xi, Y\rangle\langle X, \varphi Z\rangle-\langle\xi, X\rangle\langle Y, \varphi Z\rangle+2\langle\xi, Z\rangle\langle X, \varphi Y\rangle\}.$$

§ 3. Totally umbilical hypersurfaces.

It is well known that a totally umbilical hypersurface M in a Riemannian manifold $\tilde{M}$ is parallel if and only if the mean curvature ρ of M in $\tilde{M}$ is a constant.

PROPOSITION 1 ([8]). *Let M be a totally umbilical hypersurface in a Sasakian space form $\tilde{M}(c)$ of dimension $2m+1(\geq 5)$. Then $c=1$ and M is parallel.*

PROOF. Since M is totally umbilical, we have

(3. 1) $$(\nabla_X A)Y=(X\rho)Y$$

for any vectors X, Y tangent to M. By using the equation of Codazzi (2.13), we get

(3. 2) $$(X\rho)Y-(Y\rho)X=\frac{c-1}{4}\{f(\eta(Y)X-\eta(X)Y)+\langle\xi, X\rangle\varphi Y-\langle\xi, Y\rangle\varphi X$$
$$+2\langle X, \varphi Y\rangle\xi\}$$

for any vectors X, Y tangent to M. We can choose a nonzero vector X tangent to M_o such that $\langle\xi, X\rangle=\eta(X)=0$ as dim $\tilde{M}(c)\geq 5$. Substituting $Y=\xi$ into (3. 2) and taking the inner product with φX, we obtain

(3. 3) $$(c-1)(1-f^2)\|X\|^2=0.$$

Since $M_o:=\{x\in M \mid (1-f^2)(x)\neq 0\}$ is nonempty, (3. 3) implies $c=1$.

From this and (4. 2), we have

(3. 4) $$(X\rho)Y=(Y\rho)X$$

for any vectors X, Y tangent to M. For any tangent vector X of M, we can choose a nonzero tangent vector Y such that $\langle X, Y\rangle=0$. Thus we obtain

(3. 5) $$X\rho=0$$

for any vector X tangent to M, i. e., ρ is a constant. Therefore M is parallel. Q. E. D.

COROLLARY 2. *There are no totally umbilical hypersurfaces in a Sasakian space form* $\tilde{M}(c)$, $c\neq 1$, *of dimension* $2m+1(\geq 5)$.

From (2. 13), we can easily see the following

REMARK. If there exists a parallel hypersurface in a Sasakian space form $\tilde{M}(c)$ of dimension $2m+1(\geq 5)$, then $c=1$.

§ 4. Einsten hypersurfaces.

In this section, we prove the following

THEOREM 3. *There are no Einstein hypersurfaces in a Sasakian space form* $\tilde{M}(c)$, $c\neq 1$, *of dimension* $2m+1(\geq 5)$.

PROOF. Let M be a Einstein hypersurface in $\tilde{M}(c)$, i. e.,

$$(4.\ 1)\qquad \langle SX, Y\rangle=\frac{r}{2m}\langle X, Y\rangle,$$

where S is the Ricci tensor and r the scalar curvature of M. From the Gauss equation (2. 12) and (4. 1), we have

$$(4.\ 2)\qquad A^2X=h_1AX+\frac{1}{4}\{4(2m-1)+(2m+1+f^2)(c-1)-\frac{2r}{m}\}X-\frac{3}{4}(c-1)\langle\xi, X\rangle\xi$$
$$-\frac{2m+1}{4}(c-1)\eta(X)\xi,$$

where $h_1:=\mathrm{tr}A$.

On the other hand, from (2. 12), we have

$$(4.\ 3)\qquad r=2m(2m-1)+(m+1)(m-1+f^2)(c-1)+(h_1)^2-h_2,$$

where $h_2:=\mathrm{tr}A^2$.

Substituting (4. 3) in (4. 2), we obtain

$$(4.\ 4)\qquad A^2X=h_1AX+\frac{1}{4m}\{(m+2)(c-1)(1-f^2)-2((h_1)^2-h_2)\}X-\frac{3}{4}(c-1)\langle\xi, X\rangle\xi$$
$$-\frac{2m+1}{4}(c-1)\eta(X)\xi.$$

Since A is symmetric and $c\neq 1$, from (4. 4), we have

$$(4.\ 5)\qquad ((2m+1)\{\eta(X)A\xi-\langle A\xi, X\rangle\xi\}=3\{\langle A\xi, X\rangle\xi-\langle\xi, X\rangle A\xi\},$$

from which we obtain

$$(4.\ 6)\qquad (1-f^2)A\xi=\langle A\xi, \xi\rangle\xi+\frac{2m+1}{3}\langle A\xi, \xi\rangle\xi$$

and

$$(4.7)\qquad (1-f^2)A\xi=\frac{3}{2m+1}\langle A\xi,\xi\rangle\xi+\langle A\xi,\xi\rangle\xi.$$

Therfore ξ and ξ are principals on $M_o:=\{x\in M\mid (1-f^2)(x)\neq 0\}$, because dim $M\geq 4$. Note that f is a nonconstant function on M_o. In fact, if f is constant, then $A\xi=\xi$ because of (2.11).

Henceforth we consider M_o. We put $A\xi=\alpha\xi$ and $A\xi=\beta\xi$, where α and β are functions on M_o. These functions are called the principal curvatures. Using (2.13), we obtain

$$\begin{aligned}(\nabla_\xi A)X&=(\nabla_X A)\xi+\frac{c-1}{4}\{3f\eta(X)\xi+f\langle\xi,X\rangle\xi+(1-f^2)\varphi X\}\\&=(X\alpha)\xi+\alpha(\varphi AX-fX)-A\varphi AX+fAX\\&\quad+\frac{c-1}{4}\{3f\eta(X)\xi+f\langle\xi,X\rangle\xi+(1-f^2)\varphi X\}.\end{aligned}$$

Substituting (4.8) in $\langle(\nabla_\xi A)X,Y\rangle=\langle X,(\nabla_\xi A)Y\rangle$, we have

$$\begin{aligned}&(X\alpha)\xi-\langle\xi,X\rangle\mathrm{grad}\alpha+\alpha(\varphi A+A\varphi)X-2A\varphi AX\\&=-\frac{c-1}{2}\{f\eta(X)\xi-f\langle\xi,X\rangle\xi+(1-f^2)\varphi X\}.\end{aligned}\qquad(4.9)$$

Aplying the same argument for $A\xi=\beta\xi$, we obtain

$$(4.10)\qquad(\varphi A+A\varphi)X=2\beta\varphi X+(X\beta)\xi-\eta(X)\mathrm{grad}\beta,$$

from which we have

$$(4.11)\qquad(1-f^2)\mathrm{grad}\beta=(\beta-\alpha)f\xi+(\xi\beta)\xi.$$

Therefore we may rewrite (4.10) as:

$$(4.12)\qquad(\varphi A+A\varphi)X=2\beta X-\frac{f(\beta-\alpha)}{1-f^2}\{\eta(X)\xi-\langle\xi,X\rangle\xi\}$$

Since gradβ is a linear combination of ξ and ξ, we have

$$(4.13)\qquad(1-f^2)\mathrm{grad}\beta=(\xi\beta)\xi+(\xi\beta)\xi.$$

Comparing (4.11) whth (4.13), we get

$$(4.14)\qquad\xi\beta=(\beta-\alpha)f.$$

With (4.9) and (4.13),

$$(4.15)\qquad(1-f^2)\mathrm{grad}\alpha=(\xi\alpha)\xi+f\{\alpha(\beta-\alpha)-(c-1)(1-f^2)\}\xi,$$

from which

$$(4.16)\qquad\xi\alpha=\{\alpha(\beta-\alpha)-(c-1)(1-f^2)\}f.$$

Using (4.4), (4.9), (4.14) and (4.15), we obtain

$$(4.17)\qquad (h_1-2\beta)AX=\frac{1}{2m}\{2m\alpha\beta+(m+1)(c-1)(1-f^2)-(h_1)^2+h_2\}X$$
$$+\{\frac{\alpha(\beta-\alpha)}{1-f^2}-(c-1)\}<\xi, X>\xi+\{\frac{\beta(\beta-\alpha)}{1-f^2}-\frac{m+1}{2}(c-1)\}\eta(X)\xi,$$

from which we get

$$(4.18)\qquad 2\beta h_1-h_2=2m\alpha\beta+\beta^2-\alpha^2+\frac{m-1}{2}(c-1)(1-f^2).$$

Here we put $M_1:=\{x\in M_o \mid (h_1-2\beta)(x)\neq 0\}$ and examine two cases.

Case (I). $M_1=\phi$, i. e., $h_1=2\beta$ on M_o.

From (4. 17), we have

$$(4.19)\qquad h_2=4\beta^2-2m\alpha\beta-(m+1)(c-1)(1-f^2),$$
$$(4.20)\qquad \alpha(\beta-\alpha)=(c-1)(1-f^2)$$

and

$$(4.21)\qquad 2\beta(\beta-\alpha)=(m+1)(c-1)(1-f^2).$$

From (4. 20) and (4. 21), we have

$$(4.22)\qquad (\beta-\alpha)\{2\beta-(m+1)\alpha\}=0.$$

Therefore

$$(4.23)\qquad 2\beta=(m+1)\alpha,$$

as $c\neq 1$ and $f^2<1$.

Using (4. 19), (4. 21) and (4. 23), we get

$$(4.24)\qquad 0\leq 2h_2=(m-3)(m+1)\alpha^2.$$

When $m\geq 3$, from (4. 24), h_2 must vanish on M_o, that is, M_o is totally geodesic.

When $m=2$, we have

$$(4.25)\qquad h_1=3\alpha\,(=2\beta)\ \textit{and}\ h_2=\frac{3}{2}\alpha^2\,(=\frac{2}{3}\beta^2=\alpha\beta).$$

Therefore other principal curvatures λ and μ must satisty the quadratic equation

$$t^2-\frac{1}{2}\alpha t+\alpha^2=0.$$

As λ and μ are real numbers we have

$$\alpha=\beta=\lambda=\mu=0,$$

i. e., M_o is totally geodesic. By virtue of Corollary 2, this is a contradiction.

Case (II). $M_1\neq\phi$.

We consider M_1. In this case, we have

$$(4.26)\qquad AX=\frac{h_1-\alpha-\beta}{2(m-1)}X+\frac{(2m-1)\alpha+\beta-h_1}{2(m-1)(1-f^2)}\langle\xi, X\rangle\xi+\frac{\alpha+(2m-1)\beta-h_1}{2(m-1)(1-f^2)}\eta(X)\xi.$$

Comparing (4.18) and (4.26), we have

$$(4.27)\qquad m(h_1-2\beta)(\alpha+\beta-h_1)=(m-1)\{2m\alpha\beta+(m+1)(c-1)(1-f^2)-(h_1)^2+h_2\},$$

$$(4.28)\qquad (2\beta-h_1)\{(2m-1)\alpha+\beta-h_1\}=2(m-1)\{\alpha\beta-\alpha^2-(c-1)(1-f^2)\}$$

and

$$(4.29)\qquad (2\beta-h_1)\{\alpha+(2m-1)\beta-h_1\}=2(m-1)\{\beta^2-\alpha\beta-\frac{m+1}{2}(c-1)(1-f^2)\}.$$

From (4.28) and (4.29), we have

$$(4.30)\qquad 2(\beta-\alpha)(\alpha+\beta-h_1)=-(m-1)(c-1)(1-f^2).$$

Here the right hand side of (4.30) never vanishes. From (4.18) and (4.30), we obtain

$$(4.31)\qquad h_2=(\alpha+\beta)h_1-2m\alpha\beta.$$

Combining (4.27), (4.30) and (4.31), we get

$$(4.32)\qquad (\alpha+\beta-h_1)\{2(m+1)\alpha-2\beta-h_1\}=0,$$

from which

$$(4.33)\qquad h_1=2\{(m+1)\alpha-\beta\}.$$

Substituting (4.33) into (4.26), (4.30) and (4.31) respectively, we have

$$(4.34)\qquad AX=\frac{(2m+1)\alpha-3\beta}{2(m-1)}X+\frac{3(\beta-\alpha)}{2(m-1)(1-f^2)}\langle\xi, X\rangle\xi+\frac{(2m+1)(\beta-\alpha)}{2(m-1)(1-f^2)}\eta(X)\xi,$$

$$(4.35)\qquad 2(\beta-\alpha)\{(2m+1)\alpha-\beta\}=(m-1)(c-1)(1-f^2)$$

and

$$(4.36)\qquad h_2=2\{(m+1)\alpha^2-\beta^2\}.$$

Substituting (3.33),……, (3.36) into (4.4), we obtain

$$(4.37)\qquad (\beta-\alpha)^2\{(1-f^2)X-\langle\xi, X\rangle\xi-\eta(X)\xi\}=0.$$

This is a contradiction because of $c\neq 1$, $f^2<1$ and $m\geq 2$. Q. E. D.

We have the following

CONJECTURE. There are no hypersurfaces with parallel Ricci tensor in a Sasakian space form $\tilde{M}(c)$, $c\neq 1$.

References

[1] Blair, D. E., *Contact Manifolds in Riemannian Cemetry*, Springer-Verlag (1976).

[2] Hasegawa, I., Cyclic parallel hypersurfaces in a Sasakian space form, to appear.

[3] Ki, U-H., Real hypersurfaces with parallel Ricci tensor of a complex space form, Tsukuba J. Math., 13(1989), 73-81.

[4] Kon, M., Pseudo-Einstein real hypersurfaces in complex space forms, J. Differential Geom., 14(1979), 339-354.

[5] Montiel, S., Real hypersurfaces of a comlex hyperbolic space, J. Math. Soc. Japan, 37(1985), 515-535.
[6] Tanno, S., Sasakian manifolds with constant φ-holomorphic sectional curvature, Tôhoku Math. J., 21(1969), 501-507.
[7] Tashiro, Y. and Tachibana, S., On Fubinian and C-Fubinian manifolds, Kōdai Math. Sem. Rep., 15(1963), 176-183.
[8] Yamaguchi, S., On hypersurfaces in Sasakian manifolds, Kōdai Math. Sem. Rep., 21(1969), 64-72.
[9] Yano, K. and Okumura, M., On (f, g, u, v, λ)-structures, Kōdai Math. Sem. Rep., 22(1970), 401-423.
[10] Yano, K. and Kon, M., *CR Submanifolds of Kaehlerian and Sasakian Manifolds*, Birkhäuser(1983).