ANALYTIC MAPPING AND GREEN CAPACITY

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ABSTRACT

Let C be the Green capacity on a hyperbolic Riemann surface R. Let ϕ be a nonconstant analytic mapping of a hyperbolic Riemann surface R into a hyperbolic Riemann surface R'. Then $C(K) \geqq C(\phi(K))$ holds for any compact subset K of R.

PRELIMINARIES

We shall use the same notations as in [1], for instance, the Green function g_z^R of a hyperbolic Riemann surface R with the pole at z, the Green potential p^μ associated with a measure μ, the equilibrium measure κ^K of a compact set K, the Green capacity C, etc.

MAIN RESULTS

We shall prove

Theorem. *Let ϕ be a nonconstant analytic mapping of a hyperbolic Riemann surface R into a hyperbolic Riemann surface R'. If K is a compact subset of R, then $C(K) \geqq C(\phi(K))$.*

Proof. We may assume K is a regular compact subset of R. Set $K' = \phi(K)$. We consider the following positive linear functional T on $BC(K')$, where $BC(K')$ means the space of all bounded continuous functions on K'. For each $f \in BC(K')$ we set

$$T_f = \int (f \circ \phi)(z)\, d\kappa^K(z).$$

By Riesz representation theorem, there exists a positive Radon measure μ on K' such that

$$T_f = \int f\, d\mu \quad \text{for each } f \in BC(K').$$

It follows that

$$\int f\,d\mu = \int (f\circ\phi)(z)\,d\kappa^K(z) \quad \text{for each Borel measurable function } f \text{ on } R'.$$

For each point z on K,

$$\begin{aligned} p^\mu(\phi(z)) &= \int g^{R'}_{\phi(z)}\,d\mu \\ &= \int (g^{R'}_{\phi(z)}\circ\phi)(\zeta)\,d\kappa^K(\zeta) \\ &= \int g^{R'}_{\phi(z)}(\phi(\zeta))\,d\kappa^K(\zeta). \end{aligned}$$

From Lindelöf principle,

$$g^{R'}_{\phi(z)}(\phi(\zeta)) \geqq g^R_z(\zeta)$$

Therefore

$$p^\mu(\phi(z)) \geqq \int g^R_z(\zeta)\,d\kappa^K(\zeta) = p^{\kappa^K}(z) = 1$$

and $p^\mu\circ\phi \geqq 1$ on K. Hence $p^\mu \geqq 1$ on K' and $p^\mu \geqq p^{\kappa^{K'}}$ on K'. Hence

$$\mu(R') \geqq \kappa^{K'}(R')$$

and

$$\begin{aligned} C(K') &= \kappa^{K'}(K') = \kappa^{K'}(R') \\ &\geqq \mu(R') = \int d\mu = \int d\kappa^K = \kappa^K(R) = \kappa^K(K) = C(K), \end{aligned}$$

which completes the proof. □

References

[1] C. Constantinescu and A. Cornea, Ideale Ränder Riemannscher Flächen., Springer, Berlin-Göttingen-Heiderberg., 1963.