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北海道教育大学紀要（第2部A）第44巻　第1号　　平成5年7月
Journal of Hokkaido University of Education (Section II A) Vol.44, No.1　　July, 1993

Light-Front Tamm-Dancoff Treatment of Photons

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ABSTRACT

This is a brief review of the discussion of the mass renormalization for a photon in Light-Front Tamm-Dancoff approximations in QED. The relativistic mass counter term, independent of external momenta and free from infrared cut-off parameters, for a photon are obtained by transverse dimensional regularization. The results coincide with perturbative results.

This letter briefly review, renormalization of the mass divergence for a photon in 3 + 1 dimensional Light-Front Tamm-Dancoff approximated (LFTD) QED. The calculations, which will be provided below, coincide with those in Light Front Perturbation Theory (LFPT) [1]. The aim of this letter is to demonstrate that LFTD works well with relativistic bound state problems.

The LFTD was proposed by Perry *et al.* [2] as an alternative to lattice gauge theory. It is expected to provid a powerful tool to enable treatment of relativistic bound states.

The LFTD is the Tamm-Dancoff approximation [3] [4] applied to light front field theory instead of the usual field theory quantized in the Lorentz frame.

It is necessary to study renormalization in LFTD approximation before proceeding to practical calculations. The renormalization procedure in LFTD has been extensively studied for 1 + 1 dimensional Yukawa theory ($Yukawa_{1+1}$) [5]. In 1 + 1 dimensional space-time, there are “self-induced inertia terms” [6] which arise from the normal ordering procedure of the interaction Hamiltonian, and self-induced inertia seem to play a crucial role in $Yukawa_{1+1}$. In $3+1$ dimensional space-time, self-induced inertia vanishes when we utilize the transverse dimensional regularization method [1] which is introduced to regularize ultra-violet divergence. The results in $Yukawa_{1+1}$ do not directly apply to practical theories, such as $3+1$ dimensional LFTD QED or LFTD QCD, and to show that the mass counter terms enable finite and relativistic equations for bound states.

In the light front coordinate system, Hamiltonian is given as follows.

$$P^- = P_0^- + P_I^-. \tag{1}$$

Here P_0^- and P_I^- are free- and interaction Hamiltonians, respectively, and are defined by the use of H_0, V_1, V_2, and V_3, which are given in Refs. [1] and [7], as

$$P_0^- = H_0 + V_2 - :V_2:,\ P_I^- = V_1 + :V_2: + V_3. \tag{2}$$

Note that $:V_1: = V_1$ and $:V_3: = V_3$ and explicit form of V_1, V_2 and V_3 are given in appendix. We show the explicit forms of P_0^- and P_I^- in the following.

$$\begin{aligned} P_0^- = &\sum_\lambda \int d^2k_\perp dk^+ a^\dagger(k\lambda)\, a(k\lambda) \frac{1}{2k^+}[k_\perp^2 + \alpha(k^+)] \\ &+ \sum_s \int d^2p_\perp dp^+ \frac{1}{2p^+}[b^\dagger(ps)\, b(ps)\{m^2 + p_\perp^2 + \beta(p^+)\} \\ &+ d^\dagger(ps)\, d(ps)\{m^2 + p_\perp^2 + \gamma(p^+)\}]. \end{aligned} \tag{3}$$

Here α, β and γ are defined as

$$\alpha(k^+) = \frac{e^2}{(2\pi)^3}\int d^2p_\perp dp^+\left[\frac{1}{k^+ - p^+} - \frac{1}{k^+ + p^+}\right], \tag{4}$$

$$\beta(p^+) = \frac{e^2}{(2\pi)^3}\int d^2k_\perp dk^+ \frac{p^+}{k^+(p^+ - k^+)} \tag{5}$$

and

$$\gamma(p^+) = \frac{e^2}{(2\pi)^3}\int d^2k_\perp dk^+ \frac{p^+}{k^+(p^+ + k^+)}. \tag{6}$$

α, β, and γ in these expressions are the so-called self-inertia of a photon, a fermion, and an anti-fermion, and vanish when we use the transverse dimensional regularization method

$$\int d^2p_\perp \Rightarrow \mu^{2\epsilon}\int d^Dp_\perp, \tag{7}$$

where $D = 2 - 2\epsilon$ and μ is the renormalization mass. Thus, we may neglect α, β, and γ provided that we use the transverse dimensional regularization method.
Momentum operators are defined by

$$\begin{aligned} P^i = &\sum_\lambda \int d^3k a^\dagger(k,\lambda)\, a(k,\lambda)\, k^i \\ &+ \int d^3p\{b^\dagger(p,s)\, b(p,s)\, p^i + d^\dagger(p,s)\, d(p,s)\, p^i\}, \end{aligned}$$

for $i = 1, 2$ or $+$. (8)

Because we do not use the covariant gauge but instead the light cone gauge, the photon generally acquires a mass, this paper determines how to renormalize the photon mass. Throughout this paper, we assume that the photon have a mass m, and, the first term in (9) is modified to

$$\sum_{\lambda} \int d^3k a^{\dagger}(k\lambda)\, a(k\lambda) \frac{k_{\perp}^2 + m^2}{2k^+}. \tag{9}$$

In first order LFTD approximation, the photon state $|\gamma(k_1, 1)>$ with momentum k_1 and polarization index $\lambda = 1$ is given by

$$|\gamma(k_1,1)> = d_0(k_1)\frac{a^{\dagger}(k_1,1)}{\sqrt{2k_1^+}}|0> + \frac{1}{(2\pi)^3}\sum_{r_1,r_2}\int d^3q_1 d^3q_2 \frac{1}{\sqrt{2q_1^+}\sqrt{2q_2^+}}\delta^3(k_1 - q_1 - q_2) \times$$
$$d_1(q_1,r_1;\, q_2,r_2)\, b^{\dagger}(q_1,r_1)\, d^{\dagger}(q_2,r_2)\,|0>, \tag{10}$$

where $d^{\dagger}$ is the positron creation operator. The Einstein equation is given by

$$[M^2 + P_{\perp}^2 - 2P^+P_0^-]\,|\gamma(k_1,1)> = 2P^+P_I^-|\gamma(k_1,1)>, \tag{11}$$

where M is the mass eigenvalue of the photon state. Operating $<0|\frac{a(k,1)}{\sqrt{2k^+}}$ on both sides of Eq. (11) and using the relations of the spinors given in Appendix B, we obtain

$$[M^2 - m_0^2]\,d_0(k) = \frac{e}{2(2\pi)^{9/2}}\int \frac{d^3q}{\{q^+(k^+ - q^+)\}^{3/2}} \times$$
$$[m_e k^{+2} d_1(k - q,\uparrow;\, q,\uparrow) - m_e k^{+2} d_1(k - q,\downarrow;\, q,\downarrow)$$
$$+ \{- k^+(k^+ - q^+)(q^1 + iq^2) - k^+q^+(k^1 - q^1 - ik^2 + iq^2)$$
$$+ 2k^1(k^+ - q^+)q^+\}\, d_1(k - q,\downarrow;\, q,\uparrow)$$
$$+ \{- k^+(k^+ - q^+)(q^1 - iq^2) - k^+q^+(k^1 - q^1 + ik^2 - iq^2)$$
$$+ 2k^1(k^+ - q^+)q^+\}\, d_1(k - q,\uparrow;\, q,\downarrow)], \tag{12}$$

where m_0 is the photon mass of the no-fermion sector of the Fock space.

Operating $<0|\frac{d(p_2,s_2)\, b(p_1,s_1)}{\sqrt{2p_1^+}\sqrt{2p_2^+}}$ on both sides of Eq. (11), we have

$$G(M; p_1,p_2)\, d_1(p_1,s;\, p_2,s) = \epsilon(s)\frac{(2\pi)^{3/2} e m_e (p_1^+ + p_2^+)}{\sqrt{p_1^+ p_2^+}}\, d_0(p_1 + p_2)$$
$$- \frac{2e^2(p_1^+ + p_2^+)\sqrt{p_1^+ p_2^+}}{(2\pi)^3}\int d^3q \frac{d_1(q,s;\, p_1 + p_2 - q,s)}{(q^+ - p_1^+)^2\sqrt{q^+(p_1^+ + p_2^+ - q^+)}}, \tag{13}$$

where we have used

$$G(M;p_1,p_2) = M^2 + (p_{1\perp}+p_{2\perp})^2 - (p_1^+ + p_2^+)\left\{\frac{m_e^2 + p_{1\perp}^2}{p_1^+} + \frac{m_e^2 + p_{2\perp}^2}{p_2^+}\right\}, \tag{14}$$

and

$$\epsilon(\uparrow) = 1 \text{ and } \epsilon(\downarrow) = -1, \tag{15}$$

also

$$G(M;p_1,p_2)\,d_1(p_1,s;p_2,\bar{s}) = \frac{(2\pi)^{3/2}e}{(p_1^+ + p_2^+)\sqrt{p_1^+p_2^+}} \times [-(p_1^+ + p_2^+)p_2^+(p_1^1 - i\epsilon(s)p_1^2)$$
$$-(p_1^+ + p_2^+)p_1^+(p_2^1 + i\epsilon(s)p_2^2) + 2(p_1^1 + p_2^1)p_1^+p_2^+]\,d_0(p_1+p_2)$$
$$+\frac{2e^2(p_1^+ + p_2^+)\sqrt{p_1^+p_2^+}}{(2\pi)^3}\int\frac{d^3q}{\sqrt{q^+(p_1^+ + p_2^+ - q^+)}}\times$$
$$\left[-\frac{d_1(q,s;p_1+p_2-q,\bar{s})}{(q^+ - p_1^+)^2}\right.$$
$$\left.+\frac{d_1(q,\uparrow;p_1+p_2-q,\downarrow) + d_1(q,\downarrow;p_1+p_2-q,\uparrow)}{(p_1^+ + p_2^+)^2}\right], \tag{16}$$

where

$$\bar{\uparrow} = \downarrow \text{ and } \bar{\downarrow} = \uparrow. \tag{17}$$

Substituting Eqs. (13) – (16) into Eq. (12), we get

$$[M^2 - m_0^2]\,d_0(k) = \frac{e^2}{(2\pi)^3k^+}\int\frac{d^3q}{q^{+2}(k^+ - q^+)^2}\times$$
$$\frac{4q^+(q^+ - k^+)\,(k^+q^1 - q^+k^1)^2 + k^{+2}(k^+q_\perp - q^+k_\perp)^2 + m_e^2k^{+4}}{G(M;k-q,q)}$$
$$\times\, d_0(k) + O(e^3). \tag{18}$$

Concentrating on the real photon, $M = 0$ gives

$$-m_0^2 = -\frac{e^2}{(2\pi)^3}\int_0^1\frac{dx}{x(1-x)}\int d^2q_\perp\frac{4x(x-1)\,(q'^1)^2 + (q'_\perp)^2 + m_e^2}{(q'_\perp)^2 + m_e^2}. \tag{19}$$

Here $q^+ = xk^+$ and $q'_\perp = q_\perp - xk_\perp$. Substituting $(q'^1)^2 = (q'_\perp)^2/2$ in the above integral, and applying transverse dimensional regularization method again, the infrared singularity corresponding to $x = 1$ vanishes and we have

$$\delta m_{MS}^2 = M^2 - m_{0MS}^2 = -\frac{e^2m_e^2}{4\pi^2\epsilon}. \tag{20}$$

This result coincides with the result in Ref. [1].

The renormalization procedures we have used are: i) the masses of the particles, included in the free Hamiltonian, depend on the sector of the Fock space; the masses, in the interaction Hamiltonian, is the physical mass. ii) the coefficients of the wave function must vanish on the threshold of the physical processes to determine the sector-dependent mass, sector by sector.

ACKNOWLEDGMENTS

I acknowledge useful discussions with K. G. Wilson, K. Tanaka, R. J. Perry, and A. Harindranath. I would like to thank the Ohio State University, Department of Physics, where almost all of this work was completed.

Appendix A

The following are the explicit forms of V_1, V_2 and V_3.

$$V_1 = -\frac{e}{(2\pi)^{3)2}} \int \frac{d^2p_\perp dp^+}{\sqrt{2p^+}} \frac{d^2q_\perp dq^+}{\sqrt{2q^+}} \frac{d^2k_\perp dk^+}{\sqrt{2k^+}} \sum_{s,r,\lambda} \epsilon^\lambda_\mu (k) \times$$

$$[b^\dagger(ps)\, b(qr)\, a(k\lambda)\, \bar{u}(ps)\, \gamma^\mu u(qr)\, \delta^3(p-q-k)$$

$$+ b^\dagger(ps)\, b(qr)\, a^\dagger(k\lambda)\, \bar{u}(ps)\, \gamma^\mu u(qr)\, \delta^3(p-q+k)$$

$$+ b^\dagger(ps)\, d^\dagger(qr)\, a(k\lambda)\, \bar{u}(ps)\, \gamma^\mu v(qr)\, \delta^3(p+q-k)$$

$$- b(qr)\, d(ps)\, a^\dagger(k\lambda)\, \bar{v}(ps)\, \gamma^\mu u(qr)\, \delta^3(-p-q+k)$$

$$- d^\dagger(qr)\, d(ps)\, a(k\lambda)\, \bar{v}(ps)\, \gamma^\mu v(qr)\, \delta^3(-p+q-k)$$

$$- d^\dagger(qr)\, d(ps)\, a^\dagger(k,\lambda)\, \bar{v}(ps)\, \gamma^\mu v(qr)\, \delta^3(-p+q+k)]. \quad \text{(A.1)}$$

Here, e denotes the proton charge and we have used the abbreviation $\delta^3(P) = \delta^2(P_\perp)\,\delta(P^+)$.

$$: V_2 : = \frac{e^2}{2(2\pi)^3} \int \frac{d^2p_\perp dp^+}{\sqrt{2p^+}} \frac{d^2p'_\perp dp'^+}{\sqrt{2p'^+}} \frac{d^2k_\perp dk^+}{\sqrt{2k^+}} \frac{d^2k'_\perp dk'^+}{\sqrt{2k'^+}} \sum_{s,s',\lambda,\lambda'} \times$$

$$[b^\dagger(ps)\, b(p's')\, a(k\lambda)\, a(k'\lambda')\, \frac{\bar{u}(ps)\, \not\epsilon^\lambda(k)\, \gamma^+ \not\epsilon^{\lambda'}(k')\, u(p's')}{k^+ - p^+}\, \delta^3(p-p'-k-k')$$

$$+ b^\dagger(ps)\, b(p's')\, a^\dagger(k'\lambda')\, a(k\lambda)\, \frac{\bar{u}(ps)\, \not\epsilon^\lambda(k)\, \gamma^+ \not\epsilon^{\lambda'}(k')\, u(p's')}{k^+ - p^+}\, \delta^3(p-p'-k+k')$$

$$+ b^\dagger(ps)\, d^\dagger(p's')\, a(k\lambda)\, a(k'\lambda')\, \frac{\bar{u}(ps)\, \not\epsilon^\lambda(k)\, \gamma^+ \not\epsilon^{\lambda'}(k')\, v(p's')}{k^+ - p^+}\, \delta^3(p+p'-k-k')$$

$$+ b^\dagger(ps)\, d^\dagger(p's')\, a^\dagger(k'\lambda')\, a(k\lambda)\, \frac{\bar{u}(ps)\, \not\epsilon^\lambda(k)\, \gamma^+ \not\epsilon^{\lambda'}(k')\, v(p's')}{k^+ - p^+}\, \delta^3(p+p'-k+k')$$

$$- b^\dagger(ps)\, b(p's')\, a^\dagger(k\lambda)\, a(k'\lambda')\, \frac{\bar{u}(ps)\, \not\epsilon^\lambda(k)\, \gamma^+ \not\epsilon^{\lambda'}(k')\, u(p's')}{k^+ + p^+}\, \delta^3(p-p'+k-k')$$

$$- b^\dagger(ps)\, b(p's')\, a^\dagger(k\lambda)\, a^\dagger(k'\lambda')\, \frac{\bar{u}(ps)\, \not\epsilon^\lambda(k)\, \gamma^+ \not\epsilon^{\lambda'}(k')\, u(p's')}{k^+ + p^+}\, \delta^3(p-p'+k+k')$$

$$- b^\dagger(ps)\, d^\dagger(p's')\, a^\dagger(k\lambda)\, a(k'\lambda')\, \frac{\bar{u}(ps)\, \not\epsilon^{\lambda}(k)\, \gamma^+ \not\epsilon^{\lambda'}(k')\, v(p's')}{k^+ + p^+}\, \delta^3(p + p' + k - k')$$

$$- b(p's')\, d(ps)\, a^\dagger(k'\lambda')\, a(k\lambda)\, \frac{\bar{v}(ps)\, \not\epsilon^{\lambda}(k)\, \gamma^+ \not\epsilon^{\lambda'}(k')\, u(p's')}{k^+ + p^+}\, \delta^3(-p - p' - k + k')$$

$$- d^\dagger(p's')\, d(ps)\, a(k\lambda)\, a(k'\lambda')\, \frac{\bar{v}(ps)\, \not\epsilon^{\lambda}(k)\, \gamma^+ \not\epsilon^{\lambda'}(k')\, v(p's')}{k^+ + p^+}\, \delta^3(-p + p' - k - k')$$

$$- d^\dagger(p's')\, d(ps)\, a^\dagger(k'\lambda')\, a(k\lambda)\, \frac{\bar{v}(ps)\, \not\epsilon^{\lambda}(k)\, \gamma^+ \not\epsilon^{\lambda'}(k')\, v(p's')}{k^+ + p^+}\, \delta^3(-p + p' - k + k')$$

$$+ b(p's')\, d(ps)\, a^\dagger(k\lambda)\, a(k'\lambda')\, \frac{\bar{v}(ps)\, \not\epsilon^{\lambda}(k)\, \gamma^+ \not\epsilon^{\lambda'}(k')\, u(p's')}{k^+ - p^+}\, \delta^3(-p - p' + k - k')$$

$$+ b(p's')\, d(ps)\, a^\dagger(k\lambda)\, a^\dagger(k'\lambda')\, \frac{\bar{v}(ps)\, \not\epsilon^{\lambda}(k)\, \gamma^+ \not\epsilon^{\lambda'}(k')\, u(p's')}{k^+ - p^+}\, \delta^3(-p - p' + k + k')$$

$$+ d^\dagger(p's')\, d(ps)\, a^\dagger(k\lambda)\, a(k'\lambda')\, \frac{\bar{v}(ps)\, \not\epsilon^{\lambda}(k)\, \gamma^+ \not\epsilon^{\lambda'}(k')\, v(p's')}{k^+ - p^+}\, \delta^3(-p + p' + k - k')$$

$$+ d^\dagger(p's')\, d(ps)\, a^\dagger(k\lambda)\, a^\dagger(k'\lambda')\, \frac{\bar{v}(ps)\, \not\epsilon^{\lambda}(k)\, \gamma^+ \not\epsilon^{\lambda'}(k')\, v(p's')}{k^+ - p^+}\, \delta^3(-p + p' + k + k')\Big]. \tag{A.2}$$

$$V_3 = \frac{e^2}{2(2\pi)^3} \int \frac{d^2p_\perp dp^+}{\sqrt{2p^+}} \frac{d^2p'_\perp dp'^+}{\sqrt{2p'^+}} \frac{d^2k_\perp dk^+}{\sqrt{2k^+}} \frac{d^2k'_\perp dk'^+}{\sqrt{2k'^+}} \sum_{s,s',\sigma,\sigma'} \times$$

$$\Big[- b^\dagger(p's')\, b^\dagger(k'\sigma')\, b(ps)\, b(k\sigma)\, \frac{\bar{u}(p's')\, \gamma^+ u(ps)\, \bar{u}(k'\sigma')\, \gamma^+ u(k\sigma)}{(p^+ - p'^+)^2}\, \delta^3(p' - p + k' - k)$$

$$+ b^\dagger(p's')\, b^\dagger(k'\sigma')\, d^\dagger(k\sigma)\, b(ps)\, \frac{\bar{u}(p's')\, \gamma^+ u(ps)\, \bar{u}(k'\sigma')\, \gamma^+ v(k\sigma)}{(p^+ - p'^+)^2}\, \delta^3(p' - p + k' + k)$$

$$- b^\dagger(p's')\, b(ps)\, b(k\sigma)\, d(k'\sigma')\, \frac{\bar{u}(p's')\, \gamma^+ u(ps)\, \bar{v}(k'\sigma')\, \gamma^+ u(k\sigma)}{(p^+ - p'^+)^2}\, \delta^3(p' - p - k' - k)$$

$$+ b^\dagger(p's')\, d^\dagger(k\sigma)\, b(ps)\, d(k'\sigma')\, \frac{\bar{u}(p's')\, \gamma^+ u(ps)\, \bar{v}(k'\sigma')\, \gamma^+ v(k\sigma)}{(p^+ - p'^+)^2}\, \delta^3(p' - p - k' + k)$$

$$- b^\dagger(p's')\, b^\dagger(k'\sigma')\, d^\dagger(ps)\, b(k\sigma)\, \frac{\bar{u}(p's')\, \gamma^+ v(ps)\, \bar{u}(k'\sigma')\, \gamma^+ u(k\sigma)}{(p^+ + p'^+)^2}\, \delta^3(p' + p + k' - k)$$

$$- b^\dagger(p's')\, d^\dagger(ps)\, b(k\sigma)\, d(k'\sigma')\, \frac{\bar{u}(p's')\, \gamma^+ v(ps)\, \bar{v}(k'\sigma')\, \gamma^+ u(k\sigma)}{(p^+ + p'^+)^2}\, \delta^3(p' + p - k' - k)$$

$$- b^\dagger(p's')\, d^\dagger(ps)\, d^\dagger(k\sigma)\, d(k'\sigma')\, \frac{\bar{u}(p's')\, \gamma^+ v(ps)\, \bar{v}(k'\sigma')\, \gamma^+ v(k\sigma)}{(p^+ + p'^+)^2}\, \delta^3(p' + p - k' + k)$$

$$+ b^\dagger(k'\sigma')\, b(ps)\, b(k\sigma)\, d(p's')\, \frac{\bar{v}(p's')\, \gamma^+ u(ps)\, \bar{u}(k'\sigma')\, \gamma^+ u(k\sigma)}{(p^+ + p'^+)^2}\, \delta^3(-p' - p + k' - k)$$

$$- b^\dagger(k'\sigma')\, d^\dagger(k\sigma)\, b(ps)\, d(p's')\, \frac{\bar{v}(p's')\, \gamma^+ u(ps)\, \bar{u}(k'\sigma')\, \gamma^+ v(k\sigma)}{(p^+ + p'^+)^2}\, \delta^3(-p' - p + k' + k)$$

$$+ d^\dagger(k\sigma)\, b(ps)\, d(p's')\, d(k'\sigma')\, \frac{\bar{v}(p's')\, \gamma^+ u(ps)\, \bar{v}(k'\sigma')\, \gamma^+ v(k\sigma)}{(p^+ + p'^+)^2}\, \delta^3(-p' - p - k' + k)$$

$$+ b^\dagger(k'\sigma')\, d^\dagger(ps)\, b(k\sigma)\, d(p's')\, \frac{\bar{v}(p's')\, \gamma^+ v(ps)\, \bar{u}(k'\sigma')\, \gamma^+ u(k\sigma)}{(p^+ - p'^+)^2}\, \delta^3(-p' + p + k' - k)$$

$$+ b^\dagger(k'\sigma')\, d^\dagger(ps)\, d^\dagger(k\sigma)\, d(p's') \frac{\bar{v}(p's')\gamma^+ v(ps)\, \bar{u}(k'\sigma')\gamma^+ v(k\sigma)}{(p^+ - p'^+)^2} \delta^3(-p'+p+k'+k)$$

$$- d^\dagger(ps)\, b(k\sigma)\, d(p's')\, d(k'\sigma') \frac{\bar{v}(p's')\gamma^+ v(ps)\, \bar{v}(k'\sigma')\gamma^+ u(k\sigma)}{(p^+ - p'^+)^2} \delta^3(-p'+p-k'-k)$$

$$- d^\dagger(ps)\, d^\dagger(k\sigma)\, d(p's')\, d(k'\sigma') \frac{\bar{v}(p's')\gamma^+ v(ps)\, \bar{v}(k'\sigma')\gamma^+ v(k\sigma)}{(p^+ - p'^+)^2} \delta^3(-p'+p-k'+k)].$$

(A.3)

Appendix B

In the specific representation of the γ matrices given in references [1,7], spinors are given by [7]

$$u(p,\uparrow) = \frac{1}{2^{1/4}\sqrt{p^+}} (\sqrt{2}p^+,\ p^1 + ip^2,\ m,\ 0)^t, \tag{B.1}$$

$$u(p,\downarrow) = \frac{1}{2^{1/4}\sqrt{p^+}} (0,\ m,\ -p^1 + ip^2,\ \sqrt{2}p^+)^t, \tag{B.2}$$

$$v(p,\uparrow) = -\gamma^5 u(p,\downarrow) = \frac{1}{2^{1/4}\sqrt{p^+}} (0,\ -m,\ -p^1 + ip^2,\ \sqrt{2}p^+)^t \tag{B.3}$$

and

$$v(p,\downarrow) = +\gamma^5 u(p,\uparrow) = \frac{1}{2^{1/4}\sqrt{p^+}} (\sqrt{2}p^+,\ p^1 + ip^2,\ -m,\ 0)^t. \tag{B.4}$$

Here, $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \mathbf{diag}\ (1, 1, -1, -1)$. Normalizations are

$$\bar{u}(ps)\, u(ps') = -\bar{v}(ps)\, v(ps') = 2m\delta ss', \tag{B.5}$$

$$\bar{u}(ps)\, \gamma^\mu u(ps') = \bar{v}(ps)\, \gamma^\mu v(ps') = 2p^\mu \delta ss', \tag{B.6}$$

$$\sum_{s=\uparrow,\downarrow} u(ps)\, \bar{u}(ps) = \not{p} + m, \tag{B.7}$$

and

$$\sum_{s=\uparrow,\downarrow} v(ps)\, \bar{v}(ps) = \not{p} - m. \tag{B.8}$$

The polarization vectors $\epsilon_\mu(k, \lambda)$ in the light-front coordinate and in the light cone gauge are given by [1,7]

$$\epsilon_\mu(k,1) = (k^1/k^+,\ 0,\ -1,\ 0), \tag{B.9}$$

and

$$\epsilon_\mu(k,2) = (k^2/k^+,\ 0,\ 0,\ -1), \tag{B.10}$$

A summay of useful relations of spinors:

$$\bar{u}(q,s_1)\gamma^+ u(p,s_2) = 2\sqrt{p^+q^+}\,\delta s_1 s_2, \tag{B.11}$$

$$\bar{u}(q\uparrow)\not{\epsilon}^1(k)u(p\uparrow) = \frac{-k^+p^+(q^1-iq^2)-k^+q^+(p^1+ip^2)+2k^1p^+q^+}{k^+\sqrt{p^+q^+}}, \tag{B.12}$$

$$\bar{u}(q\uparrow)\not{\epsilon}^1(k)u(p\downarrow) = \frac{m(p^+-q^+)}{\sqrt{p^+q^+}}, \tag{B.13}$$

$$\bar{u}(q\uparrow)\not{\epsilon}^2(k)u(p\uparrow) = \frac{-ik^+p^+(q^1-iq^2)+ik^+q^+(p^1+ip^2)+2k^2p^+q^+}{k^+\sqrt{p^+q^+}}, \tag{B.14}$$

$$\bar{u}(q\uparrow)\not{\epsilon}^2(k)u(p\downarrow) = \frac{-im(p^+-q^+)}{\sqrt{p^+q^+}}, \tag{B.15}$$

$$\bar{u}(q\downarrow)\not{\epsilon}^1(k)u(p\uparrow) = \frac{-m(p^+-q^+)}{\sqrt{p^+q^+}}, \tag{B.16}$$

$$\bar{u}(q\downarrow)\not{\epsilon}^1(k)u(p\downarrow) = \frac{-k^+p^+(q^1+iq^2)-k^+q^+(p^1-ip^2)+2k^1p^+q^+}{k^+\sqrt{p^+q^+}}, \tag{B.17}$$

$$\bar{u}(q\downarrow)\not{\epsilon}^2(k)u(p\uparrow) = \frac{-im(p^+-q^+)}{\sqrt{p^+q^+}}, \tag{B.18}$$

$$\bar{u}(q\downarrow)\not{\epsilon}^2(k)u(p\downarrow) = \frac{ik^+p^+(q^1+iq^2)-ik^+q^+(p^1-ip^2)+2k^2p^+q^+}{k^+\sqrt{p^+q^+}}. \tag{B.19}$$

$$\bar{v}(q,s_1)\gamma^+ u(p,s_2) = 2\sqrt{p^+q^+}\,\delta s_1\bar{s}_2, \tag{B.20}$$

$$\bar{v}(q\uparrow)\not{\epsilon}^1(k)u(p\uparrow) = \frac{m(p^++q^+)}{\sqrt{p^+q^+}}, \tag{B.21}$$

$$\bar{v}(q\uparrow)\not{\epsilon}^1(k)u(p\downarrow) = \frac{-k^+p^+(q^1+iq^2)-k^+q^+(p^1-ip^2)+2k^1p^+q^+}{k^+\sqrt{p^+q^+}}, \tag{B.22}$$

$$\bar{v}(q\uparrow)\not{\epsilon}^2(k)u(p\uparrow) = \frac{im(p^++q^+)}{\sqrt{p^+q^+}}, \tag{B.23}$$

$$\bar{v}(q\uparrow)\not{\epsilon}^2(k)u(p\downarrow) = \frac{ik^+p^+(q^1+iq^2)-ik^+q^+(p^1-ip^2)+2k^2p^+q^+}{k^+\sqrt{p^+q^+}}, \tag{B.24}$$

$$\bar{v}(q\downarrow)\not{\epsilon}^1(k)u(p\uparrow) = \frac{-k^+p^+(q^1-iq^2)-k^+q^+(p^1+ip^2)+2k^1p^+q^+}{k^+\sqrt{p^+q^+}}, \tag{B.25}$$

$$\bar{v}(q\downarrow)\not{\epsilon}^1(k)u(p\downarrow) = \frac{-m(p^++q^+)}{\sqrt{p^+q^+}}, \tag{B.26}$$

$$\bar{v}(q\downarrow)\not{\epsilon}^2(k)u(p\uparrow) = \frac{-ik^+p^+(q^1-iq^2)+ik^+q^+(p^1+ip^2)+2k^2p^+q^+}{k^+\sqrt{p^+q^+}}, \tag{B.27}$$

$$\bar{v}(q\downarrow)\not{\epsilon}^2(k)u(p\downarrow) = \frac{im(p^++q^+)}{\sqrt{p^+q^+}}. \tag{B.28}$$

$$\bar{u}(q,s_1)\not{\epsilon}^{\lambda_1}(k)\gamma^+\not{\epsilon}^{\lambda_2}(l)u(p,s_2) = 2\sqrt{p^+q^+}\,\delta s_1 s_2(i\epsilon(s_1))^{\lambda_2-\lambda_1}. \tag{B.29}$$

$$\bar{v}(q,s_1)\not{\epsilon}^{\lambda_1}(k)\gamma^+\not{\epsilon}^{\lambda_2}(l)u(p,s_2) = 2\sqrt{p^+q^+}\,\delta s_1\bar{s}_2(i\epsilon(s_2))^{\lambda_2-\lambda_1}. \tag{B.30}$$

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