On the Motions of Stars in the Galaxy

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On the Motions of Stars in the Galaxy

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福田　守：銀河系内に於ける星の運動について

Abstract

As a model of the galaxy a non-homogeneous spheroid is considered whose equidensity surfaces are similar and similarly situated spheroids. For density distribution we use the sum of two normal distribution functions whose parameters were determined so as to fit a recent radio observation. The coefficients in the expressions for stellar motions in the galaxy are calculated.

§ 1 Introduction

Many observations[1] indicate that the isophotal curves of an elliptical nebula are ellipses having a common centre and almost the same axis ratio. We therefore assume that the equidensity surfaces of the galaxy are similar ellipsoids.

In 1956, Contopoulos[2] solved the equations of motion of a star in the non-homogeneous spheroid with an arbitrary mass distribution.

On the other hand, several models[3] have been considered to represent the mass distribution of the galaxy. In 1955, Takase[4] assumed a model of the normal density distribution, and determined the parameters so as this model to represent a recent radio observation.

§ 2 The potential function

Let the equidensity ellipses be similar to a basic ellipsoid with axes a, b, and c. If a_1, b_1, and c_1 are the axes of any equidensity ellipsoid, then

$$\frac{a_1}{a}=\frac{b_1}{b}=\frac{c_1}{c}=\sqrt{u}\ .$$

A potential per unit mass at an interior point $P(x, y, z)$ of a non-homogeneous ellipsoid with density function $\sigma(u)$ is

$$U=-\pi\ Gabc\int_0^\infty\frac{d\lambda}{D}\int_\infty^u\ \sigma(u)du$$

where

$$D^2=(a^2+\lambda)(b^2+\lambda)(c^2+\lambda)$$

and

$$u=\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}+\frac{z^2}{c^2+\lambda}\ .$$

If $\sigma(u)$ can be expanded in a Taylor series we have

$$\sigma(u)=\sigma_0+\sigma_1 u+\cdots\cdots+\frac{\sigma_n}{n!}u^n+\cdots\cdots \qquad (1)$$

Now we consider the spheroid $(a=b>c)$, and then

$$u=\frac{r^2}{a^2+\lambda}+\frac{z^2}{c^2+\lambda}$$

and

$$U-U_0=-\pi G\Big[\sigma_0\Big\{A_{10}(r^2-r_0^2)+A_{01}(z^2-z_0^2)\Big\}+\cdots\cdots$$
$$+\frac{\sigma_{n-1}}{n!}\sum\frac{n!}{\mu!\nu!}A\mu\nu(r^{2\mu}z^{2\nu}-r_0^{2\mu}z^{2\nu})+\cdots\Big]$$

where $\mu+\nu=n$ and

$$A\mu\nu=a^2c\int_0^\infty\frac{d\lambda}{(a^2+\lambda)^{\mu+1}(c^2+\lambda)^{\nu+1/2}}.$$

The values of $A_{\mu\nu}$ for various values of c/a can be calculated.[2), 5)]

§ 3 Equations of motion

We consider the spheroidal system, and the projection of the attracting force on the plane of symmetry is radial. Therefore, using cylindrical co-ordinates r, θ and z, we have

$$r^2\frac{d\theta}{dt}=C \qquad (2)$$

$$\frac{C^2}{r^2}+\left(\frac{dr}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2=2U(r,\ z)+H \qquad (3)$$

$$\frac{d^2z}{dt^2}=\frac{\partial U}{\partial z}. \qquad (4)$$

The solution of system (3) and (4) can be obtained by the method of undetermined coefficients.

Let r and z be given by the series

$$\left.\begin{aligned} r&=r_0+r_1t+r_2t^2+\cdots\\ z&=z_0+z_1t+z_2t^2+\cdots\ .\end{aligned}\right\} \qquad (5)$$

Whenever $z_0=0$, we also have $z_2=0$, and from (3), (4), and (5) we find

$$r=r_0+r_1t+\left(\frac{C^2}{2r_0^3}-\pi G\sigma_0A_{10}r_0+\cdots\right)t^2+\cdots \qquad (6)$$

and

$$z=z_1t-\frac{\pi G}{3}z_1(\sigma_0A_{01}+\sigma_1A_{11}r_0^2+\cdots)t^3+\cdots\ . \qquad (7)$$

§ 4 Density distribution

TAKASE[4)] assumed following density distribution function

$$\rho(r)=\rho_1(r)+\rho_2(r)=\rho_1(0)e^{-h_1^2r^2}+\rho_2(0)e^{-h_2^2r^2} \qquad (8)$$

and determined the parameters so as the circular rotational velocity derived from this model to represent the observational result by Kwee, Muller and Westerhout,[6] who gave the rotational velocity curve over the wide range extending to the inner part of the galaxy from their 21cm observation.

Assuming $c/a=1/12$ he obtained following values :

$$h_1=0.19kpc^{-1} \qquad M_1=0.591\times10^{11}M_o$$

$$h_2=0.60kpc^{-1} \qquad M_2=0.089\times10^{11}M_o.$$

According to the relation[4]

$$M_i = \frac{\sqrt{\pi^3} \quad \sqrt{1-\varepsilon^2}}{h_i^3}\rho_i(0)$$

we find

$$\rho_1(0)=1.88\times10^8\ M_0kpc^{-3}$$

$$\rho_2(0)=8.92\times10^8\ M_0kpc^{-3}.$$

§ 5 Calculations of coefficients

Inserting $r^2=a^2u$ into the right hand side of (8) and develope those exponential functions, we have

$$\rho(u)=\rho_1(0)+\rho_2(0)-a^2\{h_1^2\rho_1(0)+h_2^2\rho_2(0)\}u+\cdots . \qquad (9)$$

From (1) and (9) we find

$$\sigma_0=\rho_1(0)+\rho_2(0)=1.08\times10^9M_okpc^{-3}$$

$$\sigma_1=-a^2\{h_1^2\rho_1(0)+h_2^2\rho_2(0)\}=-7.38\times10^{10}M_okpc^{-3} .$$

Furthermore we have $A_{10}=0.13$, $A_{01}=2.00$[5], $a^2A_{11}=1.67$[2], $a=15kpc$ and $C=2.83\times10^6$. Inserting these values into (6) and (7) we obtain

$$r=r_0+r_1t+(4.00r_0^{-3}-2.95\times10^{-14}r_0)t^2+\cdots$$

$$z=z_1t-7.00\times10^{-23}z_1(2.16\times10^9-5.84\times10^8r_0^2)t^3+\cdots$$

Giving r_0, r_1, and z_1 we can calculate the motion of the star. If we find the solution $r=r(t)$ and $z=z(t)$, θ is given by the following formula

$$\theta-\theta_0=\int_{r_0}^{r}\frac{Cdt(r)}{r^2} .$$

References

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